GEOMETRIC SERIES

A2 Unit 3: Pure Mathematics B

WJEC past paper questions: 2010 – 2017

Total marks available 118 (approximately 2 hours 20 minutes)

1. (a) A geometric series has first term a and common ratio r. Prove that the sum of the first n terms is given by
$$S_n = \frac{a(1-r^n)}{1-r}.$$
(3)
(b) The common ratio of a geometric series is positive. The sum of the first four terms of the series is 73.8. The sum to infinity of the series is 125. Find the common ratio and the first term of the geometric series. (6)

(January 10)

2. (a) Find the sum to infinity of the geometric series
$$40 - 24 + 14.4 - \dots$$
(3)
(b) Another geometric series has first term a and common ratio r. The fourth term of this geometric series is 8. The sum of the third, fourth and fifth terms of the series is 28.

(i) Show that r satisfies the equation
$$2r^2 - 5r + 2 = 0.$$
(ii) Given that $|r| < 1$, find the value r and the corresponding value of a. (6)

(Summer 10)

3. (a) The second term of a geometric series is 6 and the fifth term is 384.

(i) Find the common ratio of the series.
(ii) Find the sum of the first eight terms of the series. (6)

(b) The first term of another geometric series is 5 and the common ratio is 1.1.

(i) The nth term of this series is 170, correct to the nearest integer. Find the value of n.
(ii) Dafydd, who has been using his calculator to investigate various properties of this geometric series, claims that the sum to infinity of this series is 940. Explain why this result cannot possibly be correct. (5)

(January 11)

4. (a) A geometric series has first term a and common ratio r. Prove the sum of the first n terms is given by

$$S_n = \frac{a(1-r^n)}{1-r}.$$

   (3)

   (b) The sum to infinity of a geometric series is equal to four times the first term of the series.

   (i) Find the value of the common ratio of the series.

   (ii) Given that the sum of the first two terms of the series is 35, find the sum of the first nine terms of the series. Give your answer to the nearest whole number.   (6)

   (Summer 11)

5. (a) A geometric series has first term a and common ratio r. Prove the sum of the first n terms is given by

$$S_n = \frac{a(1-r^n)}{1-r}.$$

   (3)

   (b) The sum of the first two terms of a geometric series is 25.2. The sum to infinity of the series is 30. Given that the common ratio is positive, find the common ratio and the first term of this geometric series.   (6)

   (January 12)

6. A geometric series has first term a and common ratio r. The sum of the first and second terms of the series is 72. The sum of the first and third terms of the series is 120.

   (a) Show that r satisfies the equation

$$3r^2 - 5r - 2 = 0$$

   (4)

   (b) Given that $|r| < 1$, find the value r and the sum to infinity of the series.   (5)

   (Summer 12)

7. (a) The pth term, of a geometric series is 16. The $(p+1)$th term of this series is 24. Find the $(p+4)$th term of the series.   (3)

   (b) The sum of the first three terms of another geometric series is 22.8. The sum to infinity of the series is 18.75. Find the common ratio and the first term of this geometric series.   (6)

   (January 13)

8. (a) Find the sum of the first eighteen terms of the geometric series

$$100 + 80 + 64 + \ldots$$

   Give your answer correct to the nearest whole number.   (3)

   (b) The second term of a geometric series is -20. The sum to infinity of the series is 64.

   (i) Show that r, the common ratio of the series, satisfies the equation

$$16r^2 - 16r - 5 = 0$$

   (ii) Find the value of r, giving a reason for your answer.   (6)

   (Summer 13)

9. (a) A geometric series has first term a and common ratio r. Prove the sum of the first n terms is given by

$$S_n = \frac{a(1-r^n)}{1-r}.$$
(3)

(b) The fourth term of a geometric series is -108 and seventh term is 4.
(i) Find the common ratio of the series.
(ii) Find the sum to infinity of the series.
(6)

(January 14)

10. A geometric series has first term a and common ratio r. The sum of the second and third terms of the series is -216. The sum of the fifth and sixth terms of the series is 8.
(a) Prove that $r = -\frac{1}{3}$.
(5)
(b) Find the sum to infinity of the series.
(3)

(Summer 14)

11. (a) The eighth and ninth terms of a geometric series are 576 and 2304 respectively. Find the fifth term of the geometric series.
(3)
(b) Another geometric series has first term a and common ratio r. The third term of this geometric series is 24. The sum of the second, third and fourth terms of the series is -56.
(i) Show that r satisfies the equation
$$3r^2 + 10r + 3 = 0.$$
(ii) Given that $|r| < 1$, find the value r and the sum to infinity of the series.
(8)

(Summer 15)

12. (a) A geometric series has first term a and common ratio r. Prove the sum of the first n terms of the series is given by
$$S_n = \frac{a(1-r^n)}{1-r}.$$
(3)
(b) The sum of the first five terms of a geometric series is 275. The sum to infinity of the series is 243. Find the common ratio and the first term of the geometric series.
(6)

(Summer 16)

13. A rich businessman makes one donation per year to a certain charity. He starts by donating £100 in the first year. In each subsequent year, the value of the donation is 1.2 times the value of the previous year's donation.
(a) Find the value of the businessman's donation in the 12th year. Give your answer correct to the nearest pound.
(2)
(b) After receiving the nth donation, the charity's treasurer calculates that over the years, the businessman has donated a **total** of £15474, correct to the nearest pound. Find the value of n.
(5)

(Summer 17)