FUNCTIONS: DOMAIN, RANGE & COMPOSITE FUNCTIONS

A2 Unit 3: Pure Mathematics B

WJEC past paper questions: 2010 – 2017

Total marks available 84 (approximately 1 hour 50 minutes)

1. The functions f and g have domains $[-3, \infty)$ and $(-\infty, \infty)$ respectively and are defined by

$$f(x) = \sqrt{x+4}\ ,$$
$$g(x) = 2x^2 - 3.$$

(a) Write down the range of f and the range of g. [2]

(b) Find an expression for $gf(x)$. Simplify your answer. [2]

(c) Solve the equation $fg(x) = 17$. [4]

(Summer 10)

2. The functions f and g have domains $[0, \infty)$ and $(-\infty, \infty)$ respectively and are defined by

$$f(x) = e^x,$$
$$g(x) = 4x^3 + 7.$$

(a) Find and simplify an expression for $gf(x)$. [2]

(b) Find the domain and range of gf. [2]

(c) (i) Solve the equation $gf(x) = 18$. Give your answer correct to three decimal places.

(ii) Giving a reason, write down a value for k so that $gf(x) = k$ has no solution. [3]

(January 11)

3. The function g has domain $(-\infty, \infty)$ and is defined by

$$g(x) = \sqrt{3x^2 + 7}.$$

Solve the equation

$$gg(x) = 8.$$ [5]

(Summer 12)

4. The functions f and g have domains $(-\infty, 0)$ and $(6, \infty)$ respectively and are defined by

$$f(x) = x^2 - 19,$$
$$g(x) = 1 - \frac{1}{2}x.$$

(a) Write down the range of f and the range of g. [2]

(b) Write down the domain and range of fg. [2]

(c) (i) Write down an expression for $fg(x)$.

(ii) Hence, solve the equation

$$fg(x) = 2x - 26.$$ [4]

(Summer 11)

5. The function f has domain $[1, \infty)$ and is defined by

$$f(x) = 3x + k,$$

where k is a constant.

(a) Write down, in terms of k, the range of f. [1]

The function g has domain $[-2, \infty)$ and is defined by

$$g(x) = x^2 - 6.$$

(b) Find the least value of k so that the function gf can be formed. [2]

(c) (i) Write down an expression, in terms of k, for $gf(x)$.

(ii) Given that $gf(2) = 3$, find the value of k. [5]

(January 12)

6. The functions f and g have domains $(0, \infty)$ and $\left(0, \frac{\pi}{4}\right]$ respectively and are defined by

$$f(x) = \ln x,$$
$$g(x) = \tan x.$$

(a) (i) Write down the domain of fg.

(ii) Write down the range of fg. [3]

(b) (i) Solve the equation $fg(x) = -0.4$. Give your answer correct to two decimal places.

(ii) Giving a reason, write down a value for k so that $fg(x) = k$ has no solution. [3]

(Summer 13)

7.

(a) The functions f and g have domains $(-\infty, \infty)$ and $(0, \infty)$ respectively and are defined by

$$f(x) = x^2 - 25,$$
$$g(x) = 2x - 3.$$

(i) Write down the domain of fg.

(ii) Write down the range of fg.

(iii) Write down an expression for $fg(x)$.

(iv) Solve the equation $fg(x) = 0$. [7]

(b) The function h is defined by

$$h(x) = \frac{2x + 7}{5x - 2}.$$

(i) Show that $hh(x) = x$.

(ii) **Hence** write down an expression for $h^{-1}(x)$. [3]

(January 13)

8.

The functions f and g have domains $(0, \infty)$ and $(-\infty, -2)$ respectively and are defined by

$$f(x) = \sqrt{x^2 + 5},$$
$$g(x) = \frac{-4}{x + 1}.$$

(a) By considering $g'(x)$, show that g is an increasing function. [2]

(b) Write down the range of g. [2]

(c) Write down the domain and range of fg. [2]

(d) (i) Write down an expression for $fg(x)$.

(ii) Hence, solve the equation

$$fg(x) = 3.$$ [5]

(January 14)

9. The functions f and g have domains $[-2, \infty)$ and $[2, \infty)$ respectively and are defined by

$$f(x) = x^2 + kx - 8,$$
$$g(x) = kx - 4,$$

where k is a positive constant.

(a) Write down, in terms of k, the range of g. [1]

(b) (i) Find the least value of k so that the function fg can be formed.

(ii) Write down an expression in terms of k for $fg(x)$.

(iii) Given that $fg(3) = 0$, find the value of k. [7]

(Summer 14)

10. The function h is defined by

$$h(x) = \frac{4x+3}{5x-4}.$$

(a) Show that $hh(x) = x$. [3]

(b) **Use the result of part *(a)*** to write down an expression for $h^{-1}(x)$.
Hence evaluate $h^{-1}(-1)$. [2]

(Summer 16)

11. The function f has domain $[2, \infty)$ and is defined by

$$f(x) = 4x + k,$$

where k is a constant.

(a) Write down, in terms of k, the range of f. [1]

The function g has domain $[-3, \infty)$ and is defined by

$$g(x) = x^2 - 9.$$

(b) Find the least value of k so that the function gf can be formed. [2]

(c) (i) Write down an expression, in terms of k, for $gf(x)$.

(ii) Given that $gf(2) = 7$, find the value of k. [5]

(Summer17)