MOCK EXAM MARKSCHEME

AS Unit 2: Applied Mathematics A

Time allowed 1 hour 45 minutes **Total Marks Available 80**

Section A: Statistics 45 marks **Section B: Mechanics 35 marks**

Reminder: *Sufficient working must be shown to demonstrate the* ***mathematical*** *method employed*

Section A: Statistics

1. A researcher records the blood glucose levels of a group of females.
The results, in mmol/litre are represented in the box plot in **Fig. 2** below.

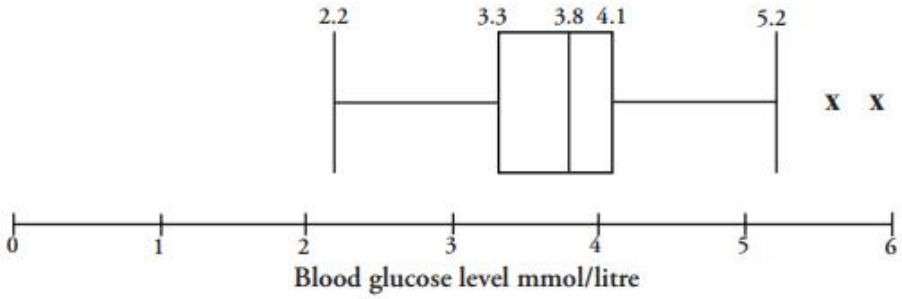


Fig.2

The researcher decides it is appropriate to clean the data by removing any outliers, which are indicated by **x** on the box plot.

If the outliers are removed from the original set of results, describe the effect this will have on

(i) the median, [1]

(ii) the standard deviation. [1]

(i) The median could remain the same or be reduced. MW1

(ii) The standard deviation will be reduced. MW1

2. **(i)** State **two** of the conditions necessary for a random variable to be modelled by a binomial distribution. [2]

Customers at a petrol station can pay by cash or by credit card.
The probability that a customer pays by cash is 0.3

(ii) 10 customers are selected at random.

Calculate the probability that exactly 2 customers pay by cash. [4]

(i) Any **two** of:

- Only 2 outcomes to each trial
- Fixed number of trials
- Probability of success the same for each trial
- Trials independent MW2

(ii) Let X be the r.v. "Number who pay by cash"
Then $X \sim B(10, 0.3)$ M1

$P(X = 2) = {}^{10}C_2 (0.3)^2 (0.7)^8$ M1 W1

$= 0.233$ W1

3. The independent events A and B are such that

$$P(A \cup B) = 0{\cdot}9, \quad P(A \cap B) = 0{\cdot}4, \quad P(A) > P(B).$$

(a) Determine the values of $P(A)$ and $P(B)$. [7]

We are given that		
$p_a \times p_b = 0.4$	**B1**	
$p_a + p_b = \mathrm{P}(A \cup B) + \mathrm{P}(A \cap B)$	**M1**	
$= 1.3$	**A1**	
$p_a + \dfrac{0.4}{p_a} = 1.3$	**M1**	
$p_a^2 - 1.3p_a + 0.4 = 0$	**m1**	
$(p_a - 0.5)(p_a - 0.8) = 0$		Or by inspection
$p_a = 0.8, p_b = 0.5$	**A1A1**	Lose A1 if wrong way round

4. Cars arrive at a car wash in such a way that the number arriving during an interval of length t minutes has a Poisson distribution with mean $0{\cdot}25t$.

(a) Find the probability that

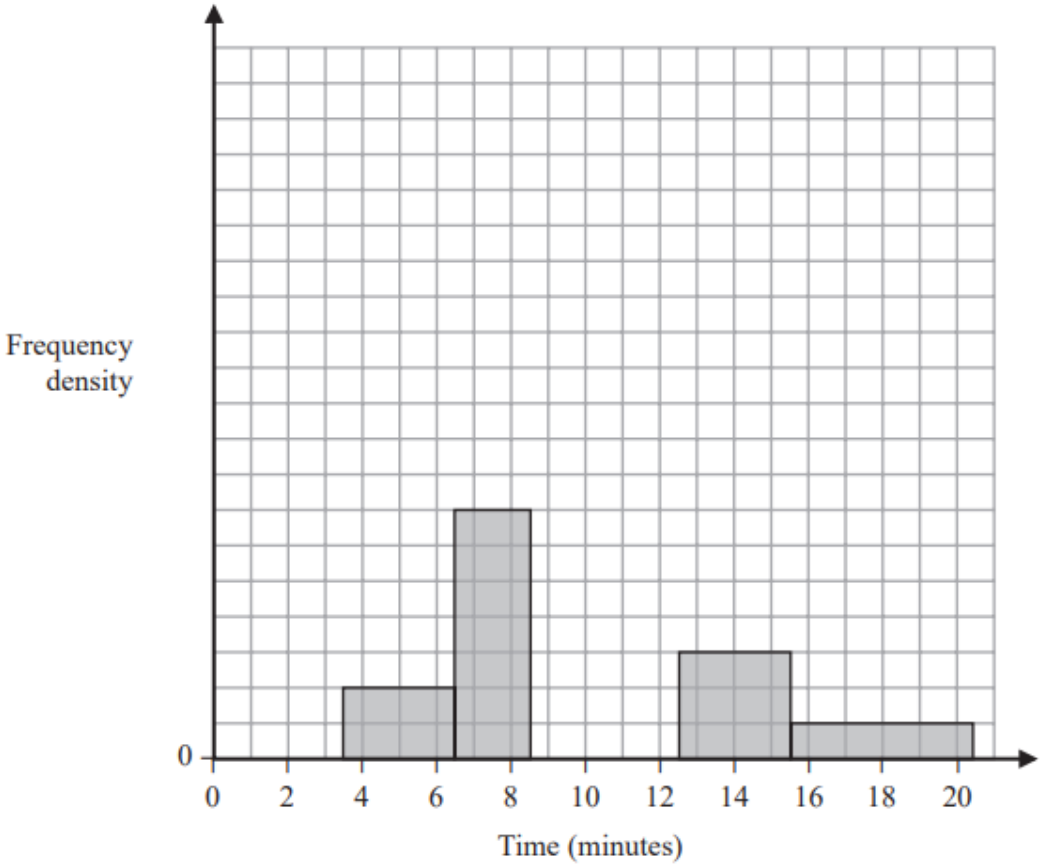
(i) exactly 4 cars arrive between 9:00 a.m. and 9:15 a.m.,

(ii) more than 6 cars arrive between 10:00 a.m. and 10:20 a.m. [6]

(b) The probability that less than 10 cars arrive during an interval of length t minutes is equal to $0{\cdot}7166$. Find the value of t. [3]

Number X arriving between 9 am and 9.15 am is Poi(3.75) si		
$P(X=4)=e^{-3.75}\times\frac{3.75^4}{4!}$	**B1**	Award M0 if tables used with mean rounded to 3.8
	M1	Award M0 if no working shown.
$=0.194$	**A1**	
Number Y arriving between 10 am and 10:20 am is Poi(5) si	**B1**	
$P(Y>6)=0.2378$	**M1A1**	M1A0 if reading adjacent row or column
Evidence of using the table in the appropriate vicinity.	**M1**	
Mean = 8	**A1**	
$t=32$	**A1**	

5. The partially completed histogram and the partially completed table show the time, to the nearest minute, that a random sample of motorists was delayed by roadworks on a stretch of motorway.



Delay (minutes)	Number of motorists
4 – 6	6
7 – 8	
9	17
10 – 12	45
13 – 15	9
16 – 20	

Estimate the percentage of these motorists who were delayed by the roadworks for between 8.5 and 13.5 minutes.

(5)

	$17+45+\frac{1}{3}\times 9 \quad [=65]$	M1	2.2a
	$(7-8)$ **14** or $(16-20)$ **5** [Values may be seen in the table]	M1 A1	3.1a 1.1b
	Percentage of motorists is $\frac{"65"}{6+"14"+17+45+9+"5"}\times 100$	M1	3.1b
	$= \mathbf{67.7\%}$	A1	1.1b
			(5 marks)

6. Sara was studying the relationship between rainfall, r mm, and humidity, h%, in the UK.
She takes a random sample of 11 days from May 1987 for Leuchars from the large data set.

She obtained the following results.

h	93	86	95	97	86	94	97	97	87	97	86
r	1.1	0.3	3.7	20.6	0	0	2.4	1.1	0.1	0.9	0.1

Sara examined the rainfall figures and found

$$Q_1 = 0.1 \qquad Q_2 = 0.9 \qquad Q_3 = 2.4$$

A value that is more than 1.5 times the interquartile range (IQR) above Q_3 is called an outlier.

(a) Show that $r = 20.6$ is an outlier.

(1)

(b) Give a reason why Sara might:

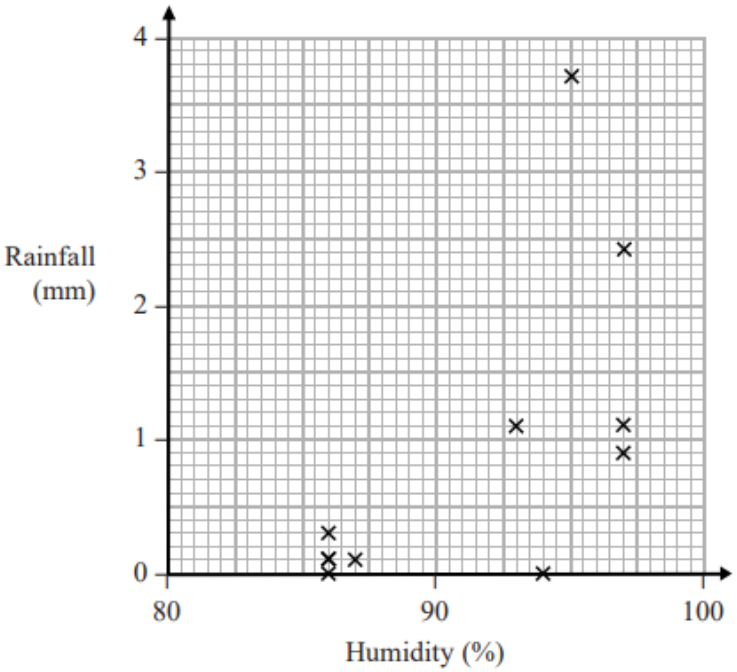
(i) include

(ii) exclude

this day's reading.

(2)

Sara decided to exclude this day's reading and drew the following scatter diagram for the remaining 10 days' values of r and h.



(c) Give an interpretation of the correlation between rainfall and humidity.

(1)

The equation of the regression line of r on h for these 10 days is r = -12.8 + 0.15*h*

(d) Give an interpretation of the gradient of this regression line.

(e) (i) Comment on the suitability of Sara's sampling method for this study. (1)

(ii) Suggest an improvement that Sara could make to her sampling method. (1)

(a)	IQR = 2.3 and 20.6 $\gg$ 2.4 + 1.5 $\times$ 2.3 (= 5.85) (Compare correct values)	B1
		(1)
(b)(i)	e.g. It is a piece of data and we should consider all the data o.e.	B1
(ii)	e.g. It is an extreme value and could unduly influence the analysis **or** It could be a mistake	B1
		(2)
(c)	e.g. "as humidity increases rainfall increases"	B1
		(1)
(d)	e.g. a 10% increase in humidity gives rise to a 1.5 mm increase in rainfall **or** represents 0.15mm of rainfall per percentage of humidity	B1
		(1)
(e)(i)	Not a good method since only uses 11 days from one location in one month	B1
(ii)	e.g. She should use data from more of the UK locations and more of the months	B1
		(2)
		(7

7. (a) The discrete random variable $X \sim B(40, 0.27)$

Find $P(X \geqslant 16)$

(2)

Past records suggest that 30% of customers who buy baked beans from a large supermarket buy them in single tins. A new manager suspects that there has been a change in the proportion of customers who buy baked beans in single tins. A random sample of 20 customers who had bought baked beans was taken.

(b) Write down the hypotheses that should be used to test the manager's suspicion.

(1)

(c) Using a 10% level of significance, find the critical region for a two-tailed test to answer the manager's suspicion. You should state the probability of rejection in each tail, which should be less than 0.05

(3)

(d) Find the actual significance level of a test based on your critical region from part (c).

(1)

One afternoon the manager observes that 12 of the 20 customers who bought baked beans, bought their beans in single tins.

(e) Comment on the manager's suspicion in the light of this observation.

(1)

Later it was discovered that the local scout group visited the supermarket that afternoon to buy food for their camping trip.

(f) Comment on the validity of the model used to obtain the answer to part (e), giving a reason for your answer.

(1)

(a)	$P(X \geqslant 16) = 1 - P(X \leqslant 15)$	M1	1.1b
	$= 1 - 0.949077\ldots$ = awrt **<u>0.0509</u>**	A1	1.1b
		(2)	
(b)	$H_0 : p = 0.3$ $H_1 : p \neq 0.3$ (Both correct in terms of p or π)	B1	2.5
		(1)	
(c)	$[Y \sim B(20, 0.3)]$ sight of $P(Y \leqslant 2) = 0.0355$ or $P(Y \leqslant 9) = 0.9520$	M1	2.1
	Critical region is $\{Y \leqslant \mathbf{2}\}$ or (o.e.)	A1	1.1b
	$\{Y \geqslant \mathbf{10}\}$ (o.e.)	A1	1.1b
		(3)	
(d)	$[0.0355 + (1 - 0.9520)] = 0.0835$ **or** **<u>8.35%</u>**	B1ft	1.1b
		(1)	
(e)	(Assuming that the 20 customers represent a random sample then) 12 is in the CR so the manager's suspicion is supported	B1ft	3.2a
		(1)	
(f)	e.g. (e) requires the 20 customers to be a random sample or independent and the members of the scout group may invalidate this so binomial distribution would not be valid (and conclusion in (e) is probably not valid)	B1	3.5a
		(1)	
			(9 marks)

Section B: Mechanics

8. *(a)* A lift, of mass 1200 kg, is moving upwards. Find the tension in the lift cable when the lift is moving with

(i) an acceleration of $2\,\text{ms}^{-2}$,

(ii) constant speed. [4]

(b) A person of mass M kg stands in a lift which is moving downwards with an acceleration of $3\,\text{ms}^{-2}$. The reaction of the floor of the lift on the person is 442 N. Determine the value of M. [3]

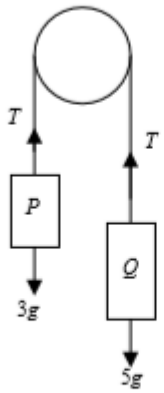
(a)(i) N2L on lift, upwards +ve	M1	dim correct, all forces T and $1200g$ opposing
$T - 1200g = 1200a$		
$T = 1200(9.8 + 0\ 2)$	A1	any correct form
$T = 14160$ (N)	A1	cao
(a)(ii) $T = 1200g$ (= 11760) (N)	B1	
(b) $Mg - R = Ma$	M1	dim correct, all forces No extra
$M(9.8 - 3) = 442$	A1	any correct form
$M = 65$	A1	cao

9. Two particles P and Q, of masses 3 kg and 5 kg respectively, are attached one to each end of a light inextensible string which passes over a smooth peg. Initially, the particles are held at rest with the string just taut and with both hanging parts of the string vertical. The particles are then released from rest.

(a) Find the magnitude of the acceleration of P and the tension in the string. [7]

(b) What assumption does the word 'light', in the description of the string, enable you to make in your solution? [1]

(a)



Apply N2L to Q	M1	$5g$ and T opposing, dim. correct
$5g - T = 5a$	A1	correct equ, allow –ve a
Apply N2L to P	M1	$4g$ and T opposing, dim. Correct
$T - 3g = 3a$	A1	correct equ consistent with first equation
Adding	m1	
$8a = 2g$		
$a = \underline{2.45\ (\text{ms}^{-2})}$	A1	cao
$T = \underline{36.75\ \text{N}}$	A1	cao

(b) Light string gives rise to tension constant throughout the string. B1

(c) Smooth peg means that the tensions in the strings on both sides of the peg are equal. B1

10. A raindrop A falls freely from rest from the top of a cliff. After it has fallen a distance 0·1 m, a second raindrop B begins to fall from rest from the top of the same cliff. The height of the cliff is 40 m.

(a) Find the velocity of A at the instant B begins to fall. [3]

(b) Find the velocity of A at the instant it reaches the ground. [2]

(c) Calculate the distance between the raindrops when the first raindrop A hits the ground. [7]

(a) Vel of A when B starts to fall

$v^2 = u^2 + 2as$, $u=0$, $a=(\pm)9.8$, $s=(\pm)0.1$ M1 oe complete method

$v^2 = 0 + 2 \times 9.8 \times 0.1$ A1

$v = \frac{7}{5}$ A1 cao

(b) Vel of A when it reaches the ground

$v^2 = u^2 + 2as$, $u=0$, $a=(\pm)9.8$, $s=(\pm)40$ M1

$v^2 = 0 + 2 \times 9.8 \times 40$

$v = 28$ A1

(c) Time of travel of B

= time for A to reach ground

$v = u + at$, $u=\frac{7}{5}$, $v=28$, $a=9.8$ M1

$28 = \frac{7}{5} + 9.8t$ A1 ft (a) and (b)

$t = \frac{19}{7}$ A1

Distance travelled by B in that time

$s = ut + \frac{1}{2}at^2$, $u=0$, $a=9.8$, $t=\frac{19}{7}$ M1

$s = 0 + \frac{1}{2} \times 9.8 \times \left(\frac{19}{7}\right)^2$ A1 ft candidates' 19/7

$s = 36.1$ A1 cao

Distance between A and $B = 40 - 36.1$

$= 3.9$ (m) A1 cao

11. Two forces $(2\mathbf{i} - 11\mathbf{j})$ N and $(6\mathbf{i} + 7\mathbf{j})$ N act on a body of mass 4 kg.

(i) Find the acceleration of the body. [3]

$\mathbf{F} = m\mathbf{a}$

$(2\mathbf{i} - 11\mathbf{j}) + (6\mathbf{i} + 7\mathbf{j}) = 4\mathbf{a}$ M1 W1

$8\mathbf{i} - 4\mathbf{j} = 4\mathbf{a}$

$2\mathbf{i} - \mathbf{j} = \mathbf{a}$ W1

12. A particle P moves in a straight line, starting from a point O. The velocity of P, measured in m s^{-1}, at time t s after leaving O is given by

$$v = 0.6t - 0.03t^2.$$

(i) Verify that, when $t = 5$, the particle is 6.25 m from O. Find the acceleration of the particle at this time. [4]

$s = 0.3t^2 - 0.01t^3$	1	**M1**	For integration
$s(5) = 0.3 \times 5^2 - 0.01 \times 5^3 = 6.25$	1	**A1**	
$a = 0.6 - 0.06t$	1	**M1**	For differentiation
$a(5) = 0.6 - 0.0 \times 5 = 0.3\ \text{ms}^{-2}$	1	**A1**	
	4		

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