

ARITHMETIC SERIES

A2 Unit 3: Pure Mathematics B

WJEC past paper questions: 2010 – 2017

Total marks available 126 (approximately 2 hours 30 minutes)

1. The sum of the first eight terms of an arithmetic series is 124 and the sum of the first twenty terms of the series is 910.
- (a) Find the first term and common difference of the series. (5)
- (b) The n th term of the series is 183. Find the value of n . (2)
- (January 10)

2. (a) An arithmetic series has first term a and common difference d .
Prove that the sum of the first n terms of the series is given by
- $$S_n = \frac{n}{2} [2a + (n - 1)d]. \quad (3)$$
- (b) The first term of an arithmetic series is 4 and the common difference is 2. The sum of the first n terms of the arithmetic series is 460. Write down an equation satisfied by n . Hence find the value of n . (3)
- (c) The fifth term of another arithmetic series is 9. The sum of the sixth term and the tenth term of this series is 42. Find the first term and the common difference of the arithmetic series. (5)
- (Summer 10)

3. (a) An arithmetic series has first term a and common difference d . Prove that the sum of the first n terms of the series is given by
- $$S_n = \frac{n}{2} [2a + (n - 1)d]. \quad (3)$$
- (b) The eighth term of an arithmetic series is 28. The sum of the first twenty terms of the series is 710. Find the first term and common difference of the arithmetic series. (5)
- (c) The first term of another arithmetic series is -3 and the fifteenth term is 67. Find the sum of the first fifteen terms of this arithmetic series. (2)
- (January 11)

4. (a) The sum of the first fifteen terms of an arithmetic series is 780. The sum of the second, fourth and tenth terms of the series is 100. Find the first term and the common difference of the series. (5)
- (b) The p th term of another arithmetic series is 1023. The $(p + 4)$ th term of this series is 1059. Find the $(p + 7)$ th term of the series. (3)
- (Summer 11)

5. The fifteenth term of an arithmetic series is seven times the fifth term.
The sum of the first eleven terms of the series is 88.
- (a) Find the first term and common difference of the arithmetic series. (6)
- (b) Given that the n th term of the series is 143, find the value of n . (2)
- (January 12)

6. (a) An arithmetic series has first term a and common difference d . Prove that the sum of the first n terms of the series is given by
- $$S_n = \frac{n}{2} [2a + (n - 1)d]. \quad (3)$$
- (b) The sum of the third, fourth and tenth terms of an arithmetic series is 79. The sum of the sixth and seventh term of the series is 61. Find the first term and the common difference of the series. (4)
- (c) Find an expression, in terms of n , for the sum of the first n terms of the arithmetic series
- $$15 + 13 + 11 + 9 + \dots$$
- Simplify your answer. (3)
- (Summer 12)

7. (a) The first term of an arithmetic series is 1 and the common difference is 4.
- (i) Show that the n th term of the arithmetic series is $4n - 3$
- (ii) The sum of the first n terms of the series is given by
- $$S_n = 1 + 5 + \dots + (4n - 7) + (4n - 3)$$
- Prove from first principles**, without using the formula for the sum of the first n terms, that
- $$S_n = n(2n - 1) \quad (4)$$
- (b) The sum of the first ten terms of another arithmetic series is 55. The sum of the fourth, seventh and ninth terms of the series is 27. Find the first term and the common difference of this arithmetic series. (5)
- (January 13)

8. (a) An arithmetic series has first terms a and common differences d . Prove that the sum of the first n terms of the series is given by
- $$S_n = \frac{n}{2} [2a + (n - 1)d]. \quad (3)$$
- (b) The sum of the first ten terms of an arithmetic series is 115. The sum of the **next** four terms of this series is 130. Find the first term and the common difference of this arithmetic series. (5)
- (Summer 13)

9. (a) The sum of the third and eighth terms of an arithmetic series is zero. The sum of the fifth, seventh and tenth terms of the series is 22. Find the first term and the common difference of the series. (4)
- (b) The first term of another arithmetic series is 9 and the common difference is 2. The sum of the first $2n$ terms of this arithmetic series is 3 times the sum of the first n terms of the series. Find the value of n . (5)
- (January 14)

10. The n th term of a number sequence is denoted by t_n . The $(n + 1)$ th term of the sequence satisfies

$$t_{n+1} = 1 - \frac{1}{t_n},$$

for all positive integers n . Given that $t_1 = 4$,

- (a) evaluate t_2, t_3 and t_4 . (2)
- (b) describe the behaviour of the sequence and hence, without carrying out any further calculation, write down the value of t_{50} . (2)
- (January 14)

11. (a) An arithmetic series has first term a and common difference d .

Prove that the sum of the first n terms of the series is given by

$$S_n = \frac{n}{2} [2a + (n - 1)d]. \quad (3)$$

- (b) The first term of an arithmetic series is 3 and the common difference is 2. The sum of the first n terms of the series is 360.

Write down an equation satisfied by n . Hence find the value of n . (3)

- (c) The tenth term of another arithmetic series is seven times the third term. The sum of the eighth and ninth terms of the series is 80. Find the first term and common difference of this arithmetic series. (4)
- (Summer 14)

12. (a) The first term of an arithmetic series is 4 and the common difference is 6.

(i) Show that the n th term of the arithmetic series is $6n - 2$.

(ii) The sum of the first n terms of the arithmetic series is given by

$$S_n = 4 + 10 + \dots + (6n - 8) + (6n - 2).$$

Without using the formula for the sum of the first n terms of an arithmetic series, **prove** that

$$S_n = n(3n + 1). \quad (4)$$

(b) The tenth term of another arithmetic series is four times the fifth term. The sum of the first fifteen terms of the series is 210.

(i) Find the first term and common difference of this arithmetic series.

(ii) Given that the k th term of the series is 200, find the value of k . (6)

(Summer 15)

13. (a) Gwilym has decided to run in a half marathon race. In order to get himself fit, he devises a training programme whereby he runs around his local track each day, gradually increasing the distance he runs from day to day. On the first day, he runs 6 laps of the track and subsequently, on any given day, he runs two laps further than he did on the day before.

(i) How many laps did he run on the 20th day of the programme?

(ii) After how many days will the total number of laps he has run since the beginning of his training programme be equal to 750? (6)

(b) The n th term of an **arithmetic** series is denoted by t_n . It is known that

$$t_{12} + t_{13} = 50.$$

(i) **Write down** the value of $t_{11} + t_{14}$. (1)

(ii) Find the sum of the first twenty-four terms of this arithmetic series. (2)

(Summer 16)

14. The n th term of a number sequence is denoted by t_n . The $(n + 1)$ th term of the sequence satisfies

$$t_{n+1} = 3t_n + 1,$$

for all positive integers n . Given that $t_4 = 202$,

(a) evaluate t_1 , (2)

(b) explain why 29 999 999 cannot be one of the terms of this number sequence. (1)

(Summer 17)

15. (a) An arithmetic series has first term a and common difference d . Prove that the sum of the first n terms of the series is given by

$$S_n = \frac{n}{2} [2a + (n - 1)d]. \quad (3)$$

(b) The sum of the first eight terms of an arithmetic series is 156 and the sum of the first sixteen terms of the series is 760. Find the first term and common difference of this series. (4)

(c) The p th term of another arithmetic series is 2057. The $(p + 5)$ th term of this series is 2102. Find the $(p + 8)$ th term of the series. (3)

(Summer 17)