PROOF BY CONTRADICTION

A2 Unit 3: Pure Mathematics B

WJEC past paper questions: 2008 – 2017

Total marks available 32 (approximately 40 minutes)

1. Prove by contradiction the following proposition.

   When x is real and positive,

$$x + \frac{49}{x} \geqslant 14 \ .$$

   The first line of the proof is given below.

   Assume that there is a positive and real value of x such that

$$x + \frac{49}{x} < 14 \ .$$ [4]

(Summer 08)

2. Complete the following proof by contradiction to show that $\sqrt{3}$ is irrational.

   Assume that $\sqrt{3}$ is rational. Then $\sqrt{3}$ may be written in the form $\frac{a}{b}$ where a and b are integers having no common factors.

   $\therefore\ a^2 = 3b^2$.
   $\therefore\ a^2$ has a factor 3.
   $\therefore\ a$ has a factor 3 so that $a = 3k$, where k is an integer. [4]

(Summer 09)

3. Prove by contradiction the following proposition.

   If a, b are positive real numbers, then $a + b \geqslant 2\sqrt{ab}$.

   The first line of the proof is given below.

   Assume that positive real numbers a, b exist such that $a + b < 2\sqrt{ab}$. [3]

(Summer 10)

4. Prove by contradiction the following proposition.

When x is real and positive,

$$4x + \frac{9}{x} \geqslant 12.$$

The first line of the proof is given below.

Assume that there is a positive and real value of x such that

$$4x + \frac{9}{x} < 12.$$ [3]

(Summer 11)

5. Complete the following proof by contradiction to show that $\sqrt{5}$ is irrational.

Assume that $\sqrt{5}$ *is rational. Then* $\sqrt{5}$ *may be written in the form* $\frac{a}{b}$ *, where a, b are integers having no common factors.*

$\therefore$ $a^2 = 5b^2$.
$\therefore$ a^2 *has a factor 5.*
$\therefore$ *a has a factor 5 so that* $a = 5k$*, where k is an integer.* [3]

(Summer 12)

6. Prove by contradiction the following proposition.

When x is real,

$$(5x - 3)^2 + 1 \geqslant (3x - 1)^2.$$

The first line of the proof is given below.

Assume that there is a real value of x such that

$$(5x - 3)^2 + 1 < (3x - 1)^2.$$ [3]

(Summer 13)

7. Complete the following proof by contradiction to show that

$$\sin\theta + \cos\theta \leqslant \sqrt{2}$$

for all values of θ.

Assume that there is a value of θ *for which* $\sin\theta + \cos\theta > \sqrt{2}$.
Then squaring both sides, we have:

[3]

(Summer 14)

8. Prove by contradiction the following proposition.

If a and b are odd integers such that 4 is a factor of $a - b$,
then 4 is **not** a factor of $a + b$.

The first lines of the proof are given below.

Assume that 4 is a factor of $a + b$.
Then there exists an integer c *such that* $a + b = 4c$. [3]

(Summer 15)

9. Prove by contradiction the following proposition.

When x is real and $x \neq 0$,

$$\left|x+\frac{1}{x}\right| \geqslant 2.$$

The first two lines of the proof are given below.

Assume that there is a real value of x *such that*

$$\left|x+\frac{1}{x}\right| < 2.$$

Then squaring both sides, we have: [3]

(Summer 16)

10. Complete the following proof by contradiction to show that $\sqrt{7}$ is irrational.

Assume that $\sqrt{7}$ *is rational. Then* $\sqrt{7}$ *may be written in the form* $\frac{a}{b}$,

where a, b *are integers having no factors in common.*

$\therefore a^2 = 7b^2$.
$\therefore a^2$ *has a factor 7.*
$\therefore a$ *has a factor 7 so that* $a = 7k$, *where* k *is an integer.* [3]

(Summer 17)