

# FURTHER INTEGRATION: SUBSTITUTION & BY PARTS

## A2 Unit 3: Pure Mathematics B

### WJEC past paper questions: 2008 – 2017

Total marks available 91 (approximately 1 hour 50 minutes)

1.

(a) Find  $\int (3x + 1) e^{2x} dx$ . [4]

(b) Use the substitution  $x = 3\sin\theta$  to show that

$$\int_{1.5}^3 \sqrt{9 - x^2} dx = \int_a^b k \cos^2 \theta d\theta$$

where the values of the constants  $a$ ,  $b$  and  $k$  are to be found.

Hence evaluate  $\int_{1.5}^3 \sqrt{9 - x^2} dx$ . [8]

(Summer 08)

2.

(a) Find  $\int (x + 3)e^{2x} dx$ . [4]

(b) Use the substitution  $u = 2\cos x + 1$  to evaluate

$$\int_0^{\frac{\pi}{3}} \frac{\sin x}{\sqrt{(2\cos x + 1)}} dx$$
[5]

(Summer 09)

3.

(a) Find  $\int x^3 \ln x dx$ . [4]

(b) Use the substitution  $u = 2x - 3$  to evaluate  $\int_1^2 x(2x - 3)^4 dx$ . [5]

(Summer 10)

4. (a) Find  $\int x \sin 2x \, dx$ . [4]

(b) Use the substitution  $u = 5 - x^2$  to evaluate

$$\int_0^2 \frac{x}{(5-x^2)^3} \, dx. \quad [4]$$

(Summer 11)

5. (a) Find  $\int x e^{-2x} \, dx$ . [4]

(b) Use the substitution  $u = 1 + 3 \ln x$  to evaluate

$$\int_1^e \frac{1}{x(1+3 \ln x)} \, dx.$$

Give your answer correct to four decimal places. [4]

(Summer 12)

6. (a) Find  $\int (3x - 1) \cos 2x \, dx$ . [4]

(b) Use the substitution  $u = 2x + 1$  to evaluate

$$\int_0^1 \frac{x}{(2x+1)^3} \, dx. \quad [5]$$

(Summer 13)

7. (a) Find  $\int x^4 \ln 2x \, dx$ . [4]

(b) Use the substitution  $u = 10 \cos x - 1$  to evaluate

$$\int_0^{\frac{\pi}{3}} \sqrt{(10 \cos x - 1)} \sin x \, dx. \quad [4]$$

(Summer 14)

8. (a) Use the substitution  $u = 12 - x^3$  to evaluate

$$\int_0^2 \frac{x^2}{(12-x^3)^2} \, dx. \quad [4]$$

(b) (i) Find  $\int x \cos 2x \, dx$ .

(ii) Use the result of (b)(i) to find

$$\int x \sin^2 x \, dx. \quad [7]$$

(Summer 15)

9.

(a) Find  $\int (2x+1)e^{-3x} dx$ . [4]

(b) Use the substitution  $u = 4 + 5 \tan x$  to evaluate

$$\int_0^{\frac{\pi}{4}} \frac{\sqrt{4+5\tan x}}{\cos^2 x} dx. \quad [4]$$

(Summer 16)

10.

(a) Find  $\int \frac{\ln x}{x^4} dx$ . [4]

(b) Use the substitution  $u = x^2 + 1$  to evaluate

$$\int_0^1 x^3(x^2 + 1)^4 dx. \quad [5]$$

(Summer 17)