VARIABLE ACCELERATION

AS Unit 2: Applied Mathematics A

Section B: Mechanics

This is a new topic at AS so there are no WJEC past paper questions. Instead, I've provided a selection of sample paper questions and exam style questions. Total marks available 34: (approximately 40 minutes)

1. A student is attempting to model the flight of a boomerang. She throws the boomerang from a fixed point O and catches it when it returns to O. She suggests the model for the displacement, s metres, after t seconds is given by

$$s = 9t^2 - \frac{3}{2}t^3, \qquad 0 \le t \le 6$$

For this model,

a) determine what happens at $t = 6$, (2)
b) find the greatest displacement of the boomerang from O, (4)
c) find the velocity of the boomerang 1 second before the student catches it, (2)
d) find the acceleration of the boomerang 1 second before the student catches it. (2)

(OCR Sample paper)

1a) at $t=6$, $s = 9(6)^2 - \frac{3}{2}(6)^3$

$s = 0$, Boomerang is back at O.

b) $v = \frac{ds}{dt} = 18t - \frac{9}{2}t^2$ (greatest displacement occurs when $v=0$)

$0 = 18t - \frac{9}{2}t^2$

$0 = 36t - 9t^2$

$0 = 9t(4-t)$, $t=0$, $t=4$

at $t=0$, $v=0$,

So must be at $t=4$

$s = 9t^2 - \frac{3}{2}t^3$

$s = 9(4)^2 - \frac{3}{2}(4)^3$

$s = 48$ metres

1c) In part a) the boomerang returns to zero displacement when $t=6$. This must mean that the student catches the boomerang when $t=6$ seconds.

∴ we require the velocity at $t=5$ seconds

$$v = 18t - \frac{9}{2}t^2$$

at $t=5$, $v = 18(5) - \frac{9}{2}(5)^2$

$$v = 90 - \frac{225}{2}$$

$$\underline{\underline{v = -22.5 \text{ ms}^{-1}}}$$

d) acceleration $= \frac{dv}{dt}$

$$= \underline{\underline{18 - 9t}}$$

at $t=5$, $a = 18 - 9(5)$

$$\underline{\underline{a = -27 \text{ ms}^{-2}}}$$

2. A particle P, of mass 3 kg, moves along the horizontal x-axis under the action of a resultant force F N. Its velocity v ms^{-1} at time t seconds is given by

$$v = 12t - 3t^2.$$

a) Given that the particle is at the origin when $t = 1$, find an expression for the displacement of the particle from O at time t s. (3)

b) Find an expression for the acceleration of the particle at time t s. (2)

(WJEC Sample paper)

2a) $v = 12t - 3t^2$

$x = \int (12t - 3t^2)\,dt$

$x = \frac{12t}{2} - \frac{3t^3}{3} + C$

$x = 6t - t^3 + C$

at $t = 1$, $x = 0$

$0 = 6 - 1 + C$, $C = -5$

$x = 6t - t^3 - 5$

b) $a = \frac{dv}{dt}$ $\quad v = 12t - 3t^2$

$\frac{dv}{dt} = 12 - 6t$

3. A particle P, of mass 400 grams, is initially at rest at the point O. The particle starts to move in a straight line so that its velocity, v ms^{-1}, at time t seconds is given by

$$v = 6t^2 - 12t^3 \quad \text{for } t > 0$$

a) Find an expression, in terms of t, for the force acting on the particle. (3)

b) Find the time when the particle next passes through O. (5)

(AQA Sample paper)

3a) Need to use 'F=ma' to find the force. So find a first.

$a = \frac{dv}{dt}$ $\quad v = 6t^2 - 12t^3$

$\frac{dv}{dt} = 12t - 36t^2$

Now, $F = ma$, where $M = 0.4$kg

$F = 0.4(12t - 36t^2)$

$F = 4.8t - 14.4t^2$

b) $v = 6t^2 - 12t^3$

$x = \int (6t^2 - 12t^3)\, dt$

$x = \frac{6t^3}{3} - \frac{12t^4}{4} + C$

given $x = 0$ when $t = 0$ (initially at rest at point O)

$0 = 0 - 0 + C$, $C = 0$

$x = 2t^3 - 3t^4$

require $x = 0$, $0 = t^3(2 - 3t)$

either $t^3 = 0$, $t = 0$ or $2 - 3t = 0$, $3t = 2$, $t = \frac{2}{3}$ s

Next passes through O at $t = \frac{2}{3}$ seconds

4. A particle moves along the horizontal x-axis so that its velocity v ms^{-1} at time t seconds is given by

$$v = 2t^2 - 3t - \frac{1}{3}t^3.$$

a) Find an expression for the acceleration of the particle at time t s. (2)
b) Find the times at which the acceleration is zero. (1)
c) Calculate the displacement of the particle from its position when $t = 1$ to its position when $t = 2$. Comment on your answer. (5)

4 a) $a = \frac{dv}{dt} = 4t - 3 - t^2$

b) require $a = 0$, $4t - 3 - t^2 = 0$

$t^2 - 4t + 3 = 0$

$(t-3)(t-1) = 0$

either $t = 1$ or $t = 3$

acceleration is zero when $t = 1$ and $t = 3$

c) $x = \int_1^2 v\,dt$

$x = \int_1^2 \left(2t^2 - 3t - \frac{1}{3}t^3\right) dt$

$x = \left[\frac{2t^3}{3} - \frac{3t^2}{2} - \frac{t^4}{12}\right]_1^2$

$x = \left[\left(\frac{16}{3} - 6 - \frac{4}{3}\right) - \left(\frac{2}{3} - \frac{3}{2} - \frac{1}{12}\right)\right] = \frac{-13}{12}$ metres

The particle travels $\frac{13}{12}$ metres in the negative x direction in the time interval 1–2 seconds.

5. A particle moves along the horizontal x-axis so that its velocity v ms^{-1} at time t seconds is given by

$$v = 9t^2 - 6t - 4.$$

At time $t = 1$, the particle's displacement from the origin is -10m. Find an expression for the displacement of the particle at time t seconds. (3)

5. $v = 9t^2 - 6t - 4$

$x = \int (9t^2 - 6t - 4)\,dt$

$x = \frac{9t^3}{3} - \frac{6t^2}{2} - 4t + c$

$x = 3t^3 - 3t^2 - 4t + c$

at $t = 1$, $x = -10$

$-10 = 3(1)^3 - 3(1)^2 - 4(1) + c$

$-10 = -4 + c$, $c = -6$

$x = 3t^3 - 3t^2 - 4t - 6$