

VECTORS

AS Unit 1: Pure Mathematics A

This is a new topic at AS so there are no WJEC past paper questions. Instead, I've provided a selection of questions for you to check your understanding.

- The vectors \mathbf{a} and \mathbf{b} are such that, $\mathbf{a} = 6\mathbf{i} - 2\mathbf{j}$ and $\mathbf{b} = 5\mathbf{i} + 3\mathbf{j}$, find
 - $\mathbf{a} - \mathbf{b}$
 - $2\mathbf{a} + 3\mathbf{b}$
 - $4\mathbf{a} - 3\mathbf{b}$
- Which of the following vectors are parallel to $2\mathbf{a} - 5\mathbf{b}$. If you find that the vectors are indeed parallel, give a geometrical interpretation of your result.
 - $6\mathbf{a} - 15\mathbf{b}$
 - $-2\mathbf{a} + 5\mathbf{b}$
 - $\mathbf{a} - 3\mathbf{b}$
 - $-\mathbf{a} + 2\mathbf{b}$
 - $0.4\mathbf{a} - \mathbf{b}$
- In the triangle XZY, $\overrightarrow{XY} = \mathbf{a}$ and $\overrightarrow{XZ} = \mathbf{b}$. The point W lies on ZY such that $ZW : WY = 1 : 4$. Find in terms of \mathbf{a} and \mathbf{b} :
 - \overrightarrow{ZY}
 - \overrightarrow{YW}
 - \overrightarrow{XW} .
- ABCD is a trapezium with AB parallel to DC. X is a point on DC such that $DX : XC = 2 : 1$ and Y is the midpoint of BC. If $\overrightarrow{AB} = 2\mathbf{b}$, $\overrightarrow{DA} = \mathbf{a}$ and $\overrightarrow{DX} = \overrightarrow{AB}$. Find in terms of \mathbf{a} and \mathbf{b} :
 - \overrightarrow{DC}
 - \overrightarrow{AY}
 - \overrightarrow{BC}
 - \overrightarrow{YX} .

5. The vectors \mathbf{a} and \mathbf{b} are such that, $\mathbf{a} = -3\mathbf{i} + 2\mathbf{j}$ and $\mathbf{b} = 4\mathbf{i} - 7\mathbf{j}$, find
 - a) $\mathbf{a} + \mathbf{b}$
 - b) $\mathbf{a} - \mathbf{b}$
 - c) $|\mathbf{a} - \mathbf{b}|$

6. In the triangle OAB, P is the mid-point of AB and Q is the point on OP such that $OQ = \frac{3}{4}OP$. Given that $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{AB} = \mathbf{b}$, find, in terms of \mathbf{a} and \mathbf{b} .
 - a) \overrightarrow{AB}
 - b) \overrightarrow{OP}
 - c) \overrightarrow{OQ}
 - d) \overrightarrow{AQ} .

7. If $\mathbf{a} = 3\mathbf{i} + 4\mathbf{j}$ and $\mathbf{b} = 2\mathbf{i} + 8\mathbf{j}$, find:
 - a) $\mathbf{a} + \mathbf{b}$
 - b) $\mathbf{a} - 2\mathbf{b}$
 - c) $|\mathbf{a} + \mathbf{b}|$
 - d) $|\mathbf{a} - 2\mathbf{b}|$

8. If the point P has position vector $2\mathbf{i} + 3\mathbf{j}$ and the point Q has position vector $7\mathbf{i} + 4\mathbf{j}$, find
 - a) \overrightarrow{PQ}
 - b) \overrightarrow{QP} .

9. The point A has position vector $3\mathbf{i} + \mathbf{j}$ and point B has position vector $10\mathbf{i} + \mathbf{j}$. Find the position vector of the point which divides AB in the ratio 3 : 4.

10. Find a unit vector in the direction of
 - a) $-5\mathbf{i} + 12\mathbf{j}$
 - b) $4\mathbf{i} - 7\mathbf{j}$

11. A has coordinates (-1, 4) and B has coordinates (5, -3).
 - a) Write down the position vectors of A and B.
 - b) Find the vector \overrightarrow{AB} .
 - c) Hence find the distance between the points A and B.

12. Two towns A and B have position vectors, relative to a certain location, of $20\mathbf{i} + 40\mathbf{j}$ and $60\mathbf{i} + 70\mathbf{j}$ respectively. Planners are considering constructing a straight motorway that would pass through the two towns, this section would form a line AB. Find the vector \overrightarrow{AB} . Calculate the length of this section of motorway.
13. John maps out the location of various places he visits in his local town. The library, L has coordinates (2, 3). The cinema, C has coordinates (5, 7) and his school, S is at (12, 8). John also notices that the park, P is exactly half-way between the library and the cinema, and his favourite restaurant R is the midpoint of CS.
- Find the position vectors of the park P and his favourite restaurant R.
 - Find \overrightarrow{LS} and \overrightarrow{PR} .
 - Find the distance between the library and John's school, LS.
 - Find the distance between the park and John's favourite restaurant, PR.
 - What do you notice about these two lines LS and PR?
14. a) The vectors \mathbf{u} and \mathbf{v} are defined by $\mathbf{u} = 2\mathbf{i} - 3\mathbf{j}$, $\mathbf{v} = -4\mathbf{i} + 5\mathbf{j}$.
- Find the vector $4\mathbf{u} - 3\mathbf{v}$
 - The vectors \mathbf{u} and \mathbf{v} are the position vectors of the points U and V, respectively. Find the length of the line UV.
- b) Two villages A and B are 40 km apart on a long straight road passing through a desert. The position vectors of A and B are denoted by \mathbf{a} and \mathbf{b} , respectively.
- Village C lies on the road between A and B at a distance 4 km from B. Find the position vector of C in terms of \mathbf{a} and \mathbf{b} .
 - Village D has position vector $\frac{2}{9}\mathbf{a} + \frac{5}{9}\mathbf{b}$. Explain why village D cannot possibly be on the straight road passing through A and B.

(Sample paper)