MOCK EXAM MARKSCHEME

AS Unit 1: Pure Mathematics A

This exam is based on 2018 Legacy WJEC C1 and C2 papers

Time allowed 2 ½ hours **Total Marks Available 121**

1. The points *A*, *B*, *C* and *D* have coordinates (–2, 7), (2, –1), (5, 3) and (3, 7) respectively.

(a) (i) Show that *AB* and *DC* are parallel.

(ii) Find the equation of *AB*. [5]

(b) The line *L* has equation $x - 2y + 11 = 0$ and intersects *AB* at the point *E*.

(i) Giving a reason, determine whether or not *L* is perpendicular to *AB*.

(ii) Show that *E* has coordinates (–1, 5).

(iii) Calculate the length of *EF*, where *F* denotes the midpoint of *AB*. [8]

(*a*) (i) Gradient of *AB*(*DC*) = $\frac{\text{increase in } y}{\text{increase in } x}$ M1

Gradient of *AB* = –2, gradient of *DC* = –2,
(or equivalent, at least one correct) A1

Gradient of *AB* = gradient of *DC* ⇒ *AB* and *DC* are parallel
(c.a.o.) A1

(ii) A correct method for finding the equation of *AB* using candidate's gradient for *AB* M1

Equation of *AB*: $y - 7 = -2\,[x - (-2)]$ (or equivalent)
(f.t. candidate's gradient for *AB*) A1

(*b*) (i) Gradient $L = {}^1/_2$ B1

$^1/_2 \times -2 = -1 \Rightarrow L$ is perpendicular to *AB* (o.e.)
(f.t. candidate's derived gradients for *AB* and *L*) B1

(ii) An attempt to solve equations of *AB* and *L* simultaneously M1

$x = -1, y = 5$ (convincing) A1

(iii) A correct method for finding the coordinates of the mid-point of *AB* M1

Mid-point of *AB* has coordinates (0, 3) A1

A correct method for finding the length of *EF* M1

$EF = \sqrt{5}$ (f.t. the candidate's derived coordinates of *F*) A1

2. Simplify $\sqrt{500}+\left(\sqrt{12}\times\sqrt{15}\right)-\dfrac{7\sqrt{60}}{\sqrt{3}}$. [4]

$\sqrt{500} = 10\sqrt{5}$		B1
$(\sqrt{12} \times \sqrt{15}) = 6\sqrt{5}$		B1
$\dfrac{7\sqrt{60}}{\sqrt{3}} = 14\sqrt{5}$		B1
$\sqrt{500} + (\sqrt{12} \times \sqrt{15}) - \dfrac{7\sqrt{60}}{\sqrt{3}} = 2\sqrt{5}$	(c.a.o.)	B1

3. The curve C has equation $y = x^2 - 6x + 7$. The point P, whose x-coordinate is 2, lies on C.

(a) Show that the equation of the **normal** to C at P is $y = \frac{1}{2}x - 2$. [6]

(b) The normal to C at P intersects C again at the point Q. Find the coordinates of Q. [4]

(*a*) y-coordinate of $P = -1$ B1

$\frac{dy}{dx} = 2x - 6$

(an attempt to differentiate, at least one non-zero term correct) M1

An attempt to substitute $x = 2$ in candidate's expression for $\frac{dy}{dx}$ m1

Value of $\frac{dy}{dx}$ at $P = -2$ (c.a.o.) A1

Gradient of normal $= \dfrac{-1}{\text{candidate's value for } \frac{dy}{dx}}$ m1

Equation of normal to C at P: $y - (-1) = {}^1/_2(x - 2)$

Equation of normal to C at P: $y = {}^1/_2 x - 2$ (convincing) A1

(*b*) $x^2 - 6x + 7 = {}^1/_2 x - 2$ M1

An attempt to collect terms, form and solve the quadratic equation in x either by correct use of the quadratic formula or by writing the equation in the form $(ax + b)(cx + d) = 0$, with $a \times c =$ candidate's coefficient of x^2 and $b \times d =$ candidate's constant m1

$2x^2 - 13x + 18 = 0 \Rightarrow (2x - 9)(x - 2) = 0$ (or equivalent)

$\Rightarrow x = {}^9/_2, (x = 2)$ (c.a.o.) A1

At Q, $x = {}^9/_2$, $y = {}^1/_4$ (c.a.o.) A1

4. *(a)* Express $4x^2 + 40x - 69$ in the form $a(x+b)^2 + c$, where the values of the constants a, b and c are to be found. [3]

(b) **Using your answer to part *(a)***, solve the equation

$$4x^2 + 40x - 69 = 0.$$ [3]

(a)	$a = 4$			B1
	$b = 5$			B1
	$c = -169$			B1
(b)	$4(x+5)^2 = 169$	(f.t. candidate's values for a, b, c)		M1
	$(x+5) = (\pm)\dfrac{13}{2}$	(f.t. candidate's values for a, b, c)		m1
	$x = \dfrac{3}{2}, -\dfrac{23}{2}$	(both values)	(c.a.o.)	A1

5. *(a)* Using the binomial theorem, write down and simplify the first four terms in the expansion of $\left(1-\frac{x}{2}\right)^7$ in ascending powers of x. [4]

(b) The coefficient of x^2 in the expansion of $(1 + 4x)^n$ is 3360. Given that n is a positive integer, find the value of n. [3]

(*a*) $\left(1-\frac{x}{2}\right)^7 = 1 - \frac{7x}{2} + \frac{21x^2}{4} - \frac{35x^3}{8} + \ldots$ B1 B1 B1 B1

(– 1 for further incorrect simplification)

(*b*) ${}^nC_2 \times 4^k = 3360$ $(k = 1, \mathbf{2})$ M1

Either $16n^2 - 16n - 6720 = 0$ or $n^2 - n - 420 = 0$ or $n(n-1) = 420$ A1

$n = 21$ (c.a.o.) A1

6. Find the range of values of x satisfying the inequality

$$9x^2 + 16x - 4 > 0.$$

[3]

Finding critical values $x = -2$, $x = {}^2/_9$ B1

A statement (mathematical or otherwise) to the effect that

$x < -2$ or $x > {}^2/_9$ (or equivalent)

(f.t. candidate's derived critical values) B2

Deduct 1 mark for each of the following errors

the use of non-strict inequalities

the use of the word 'and' instead of the word 'or'

7. *(a)* Given that $y = 9x^2 - 7x - 8$, find $\frac{dy}{dx}$ from first principles. [5]

(b) Given that $y = \frac{k}{x} + 14\sqrt{x}$ and that $\frac{dy}{dx} = 2$ when $x = 9$, find the value of the constant k. [4]

(a)	$y + \delta y = 9(x + \delta x)^2 - 7(x + \delta x) - 8$	B1
	Subtracting y from above to find δy	M1
	$\delta y = 18x\delta x + 9(\delta x)^2 - 7\delta x$	A1
	Dividing by δx and letting $\delta x \to 0$	M1
	$\frac{dy}{dx} = \lim_{\delta x \to 0} \frac{\delta y}{\delta x} = 18x - 7$	(c.a.o.) A1
(b)	$\frac{dy}{dx} = k \times (-1) \times x^{-2} + 14 \times \frac{1}{2} \times x^{-1/2}$	B1, B1
	Attempting to substitute $x = 9$ in candidate's expression for $\frac{dy}{dx}$ and putting expression equal to 2	M1
	(The M1 is only awarded if at least one B1 has been awarded)	
	$k = 27$ (c.a.o.)	A1

8. *(a)* (i) Find one real root of the equation $8x^3 + 7x^2 - 13x + 10 = 0$.

(ii) Show that the root you have found is the only real root of the equation

$$8x^3 + 7x^2 - 13x + 10 = 0.$$ [7]

(a) Denoting $8x^3 + 7x^2 - 13x + 10$ by $f(x)$,

(i) **Either:** showing that $f(-2) = 0$

Or: trying to find $f(r)$ for at least two values of r — M1

$f(-2) = 0 \Rightarrow x = -2$ is a root of $8x^3 + 7x^2 - 13x + 10 = 0$ — A1

(ii) $f(x) = (x + 2)(8x^2 + mx + n)$ with one of m, n correct — M1

$f(x) = (x + 2)(8x^2 - 9x + 5)$ — A1

An expression for $b^2 - 4ac$ for the quadratic $8x^2 - 9x + 5 = 0$ with at least two of a, b or c correct

(f.t. candidate's derived quadratic expression) — M1

$b^2 - 4ac = -79$ or $b^2 - 4ac < 0$

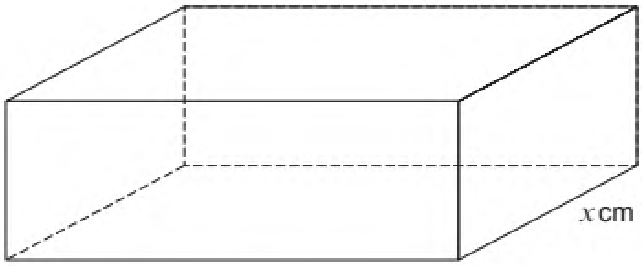
(f.t. candidate's derived quadratic expression) — A1

$b^2 - 4ac < 0 \Rightarrow 8x^2 - 9x + 5 = 0$ has no real roots $\Rightarrow x = -2$ is the only real root of $8x^3 + 7x^2 - 13x + 10 = 0$

(f.t. candidate's derived **negative** value for $b^2 - 4ac$) — A1

9

A closed box, in the form of a cuboid, is such that the length of its base is three times the width of its base. The volume of the box is 6000 cm^3. The total length of the twelve edges of the box is denoted by L cm.



(a) Show that $L = 16x + \frac{8000}{x^2}$, where x cm denotes the width of the base. [3]

(b) Find the minimum value of L, showing that the value you have found is a minimum value. [5]

(*a*) Height of box $= \frac{6000}{3x^2}$

(or an equivalent expression for the height) B1

$L = 4 \times (x + 3x + \frac{6000}{3x^2})$

(f.t. candidate's derived expression for height of box in terms of x) M1

$L = 16x + \frac{8000}{x^2}$ (convincing) A1

(*b*) $\frac{dL}{dx} = 16 - \frac{16000}{x^3}$ (o.e.) B1

Putting derived $\frac{dL}{dx} = 0$ M1

$x = 10$ (f.t. candidate's $\frac{dL}{dx}$ provided x is positive) A1

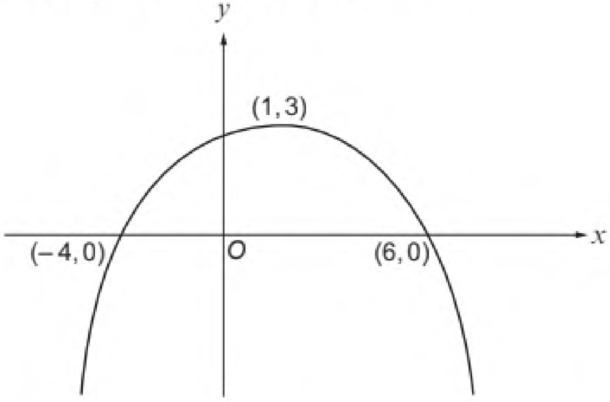
Stationary value of L at $x = 10$ is 240

(f.t. candidate's derived positive value for x) A1

A correct method for finding nature of the stationary point yielding a minimum value (provided x is positive) B1

10.

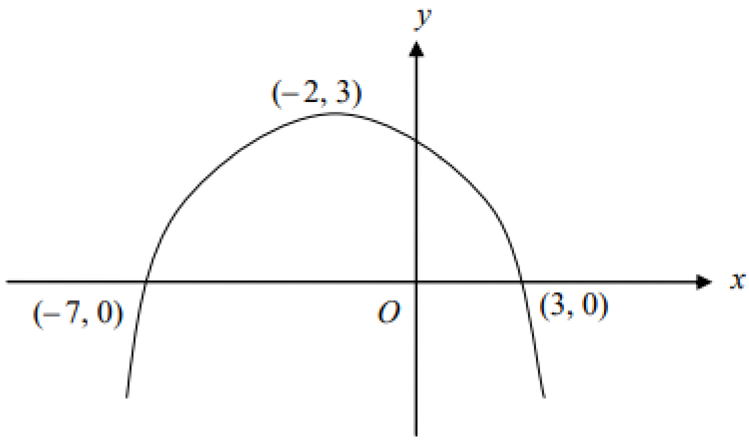
The diagram shows a sketch of the graph of $y = f(x)$. The graph passes through the points $(-4, 0)$ and $(6, 0)$ and has a maximum point at $(1, 3)$.



(a) Sketch the graph of $y = f(x + 3)$, indicating the coordinates of the stationary point and the coordinates of the points of intersection of the graph with the x-axis. [3]

(b) Gwen is asked by her teacher to draw the graph of $y = f(ax)$ for various values of the constant a. Two of Gwen's graphs pass through the point $(2, 0)$.
Find the value of a corresponding to each of these two graphs. [2]

(a)



Concave down curve with x-coordinate of maximum $= -2$	B1
y-coordinate of maximum $= 3$	B1
Both points of intersection with x-axis	B1

(b) $a = -2$ B1
$a = 3$ B1

11. (a) Find all values of θ in the range $0° \leqslant \theta \leqslant 360°$ satisfying

$$10\sin^2\theta + 3\sin\theta = 4\cos^2\theta - 2.$$ [6]

(b) Find all values of ϕ in the range $0° \leqslant \phi \leqslant 360°$ satisfying

$$\frac{3}{\cos\phi} - \frac{5}{\sin\phi} = 0.$$ [3]

(a) $10\sin^2\theta + 3\sin\theta = 4(1 - \sin^2\theta) - 2$

(correct use of $\cos^2\theta = 1 - \sin^2\theta$) M1

An attempt to collect terms, form and solve quadratic equation in $\sin\theta$, either by using the quadratic formula or by getting the expression into the form $(a\sin\theta + b)(c\sin\theta + d)$, with $a \times c$ = candidate's coefficient of $\sin^2\theta$ and $b \times d$ = candidate's constant m1

$14\sin^2\theta + 3\sin\theta - 2 = 0 \Rightarrow (2\sin\theta + 1)(7\sin\theta - 2) = 0$

$\Rightarrow \sin\theta = -\frac{1}{2}$, $\sin\theta = \frac{2}{7}$ (c.a.o.) A1

$\theta = 210°, 330°$ B1 B1

$\theta = 16{\cdot}6°, 163{\cdot}4°$ B1

Note: Subtract 1 mark for each additional root in range for each branch, ignore roots outside range.
$\sin\theta$ = +, –, f.t. for 3 marks, $\sin\theta$ = –, –, f.t. for 2 marks
$\sin\theta$ = +, +, f.t. for 1 mark

(b) Correct use of $\frac{\sin\phi}{\cos\phi} = \tan\phi$ (o.e.) M1

$\tan\phi = \frac{5}{3}$ A1

$\phi = 59°, 239°$ (f.t $\tan\phi = a$) B1

12. Find all values of x satisfying the equation

$$\log_a(11x^2 + 16x + 5) - \log_a(4x^2 + 1) = 3\log_a 2.$$ [5]

$$\log_a(11x^2 + 16x + 5) - \log_a(4x^2 + 1) = \log_a\left(\frac{11x^2 + 16x + 5}{4x^2 + 1}\right)$$

(subtraction law) B1

$3\log_a 2 = \log_a 2^3$ (power law) B1

$\dfrac{11x^2 + 16x + 5}{4x^2 + 1} = 2^3$ (removing logs) M1

An attempt to collect terms, form and solve quadratic equation with three terms in x, either by using the quadratic formula or by getting the expression into the form
$(ax + b)(cx + d)$, with $a \times c$ = candidate's coefficient of x^2 and
$b \times d$ = candidate's constant m1

$21x^2 - 16x + 3 = 0 \Rightarrow (7x - 3)(3x - 1) = 0 \Rightarrow x = {}^3/_7, {}^1/_3$

(both values, c.a.o.) A1

Note: Answer only with no working earns 0 marks

13. Given that n is an even number, prove that $9n^2 + 6n$ has a factor of 12. [3]

If n is an even number, then n can be written as $2m$

$n = 2m$

Now, $9n^2 + 6n = 9(2m)^2 + 6(2m)$ **M1**

$= 36m^2 + 12m$

$=12(3m^2 + m)$ **A1**

Hence 12 must be a factor of $9n^2 + 6n$ if n is an even number. **A1**

14

The circle C_1 has centre $A(2, -1)$ and passes through the point $P(6, 1)$.

(a) (i) Show that the equation of C_1 is given by

$$x^2 + y^2 - 4x + 2y - 15 = 0.$$

(ii) Given that the point Q is such that PQ is a diameter of C_1, find the coordinates of Q.

(iii) Find the equation of the tangent to C_1 at P. [9]

(b) The circle C_2 has centre $B(-4, 7)$ and radius $\sqrt{8}$. Find the shortest distance between C_1 and C_2. Give your answer correct to one decimal place. [3]

(a) (i) $r^2 = (6-2)^2 + (1-(-1))^2$ — B1

Equation of C_1: $(x-2)^2 + (y-(-1))^2 = 20$

(f.t. candidate's derived value for r^2) — M1

Equation of C_1: $x^2 + y^2 - 4x + 2y - 15 = 0$

(convincing) — A1

(ii) A correct method for finding Q — M1

$Q(-2, -3)$ — A1

(iii) Gradient $AP = \dfrac{\text{inc in } y}{\text{inc in } x}$ — M1

Gradient $AP = \dfrac{1-(-1)}{6-2} = \dfrac{1}{2}$ (o.e.) — A1

Use of $m_{\tan} \times m_{\text{rad}} = -1$ — M1

Equation of tangent is:

$y - 1 = -2(x - 6)$ (f.t. candidate's gradient for AP) — A1

(b) Distance between centres of C_1 and $C_2 = 10$ — B1

Use of the fact that the shortest distance between the circles = distance between centres – sum of the radii — M1

Shortest distance between the circles $= 10 - \sqrt{8} - \sqrt{20} = 2\cdot7$

(f.t. candidate's radius for C_1 and their distance between centres, provided the answer is positive) — A1

15. The points *A, B* and *C* have position vectors -2**i** + **j**, 2**i** + 5**j** and 6**i** + 3**j** respectively.

M is the midpoint of *BC.*

(a) Find the position vector of the point D such that $\overrightarrow{BC} = \overrightarrow{AD}$. [3]

(b) Find the magnitude of $\overrightarrow{AM}$. [3]

(a) $\overrightarrow{BC} = 4\mathbf{i} - 2\mathbf{j} = \overrightarrow{AD}$ **B1**

$\overrightarrow{AD} = \overrightarrow{OD} - \overrightarrow{OA}$

$4\mathbf{i} - 2\mathbf{j} = \overrightarrow{OD} - (-2\mathbf{i} + \mathbf{j})$ **M1**

$\overrightarrow{OD} = 2\mathbf{i} - \mathbf{j}$ **A1**

(b) $\overrightarrow{OM} = 4\mathbf{i} + 4\mathbf{j}$ **B1**

$\overrightarrow{AM} = \overrightarrow{OM} - \overrightarrow{OA} = 4\mathbf{i} + 4\mathbf{j} - (-2\mathbf{i} + \mathbf{j})$

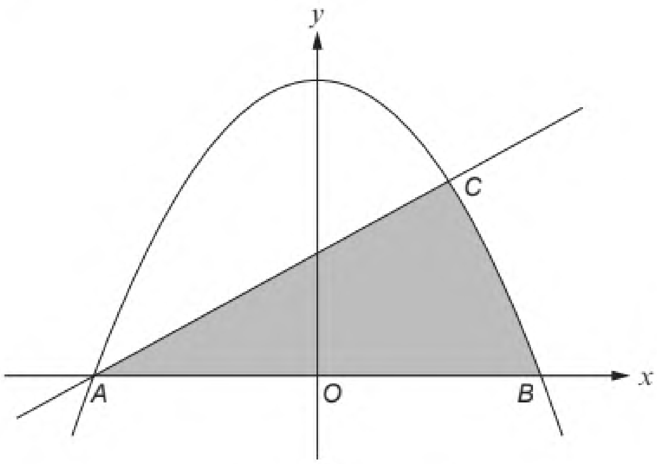
$\overrightarrow{AM} = 6\mathbf{i} + 3\mathbf{j}$ **M1**

$|\overrightarrow{AM}| = \sqrt{6^2 + 3^2} = \sqrt{45} = 3\sqrt{5}$ **A1**

16.

(*a*) Find $\int \left(\sqrt[3]{x} - \frac{4}{x^{\frac{7}{2}}} \right) \mathrm{d}x$. [2]

(*b*)



The diagram shows a sketch of the curve $y = 25 - x^2$ and the line $y = 2x + 10$. The curve and the line intersect at the points *A* and *C*. The curve intersects the x-axis at the points *A* and *B*. The coordinates of *A*, *B* and *C* are (– 5, 0), (5, 0) and (3, 16) respectively. Find the area of the shaded region. [6]

(*a*) $\frac{x^{4/3}}{4/3} - 4 \times \frac{x^{-5/2}}{-5/2} + c$ (–1 if no constant term present) B1, B1

(*b*) Use of integration to find the area under the curve M1

$\int 25\mathrm{d}x = 25x$ $\int x^2\, \mathrm{d}x = (1/3)x^3$ (both correct) A1

Correct method of substitution of candidate's limits m1

$[25x - (1/3)x^3]_3^5 = (125 - 125/3) - (75 - (27/3)) = 52/3$

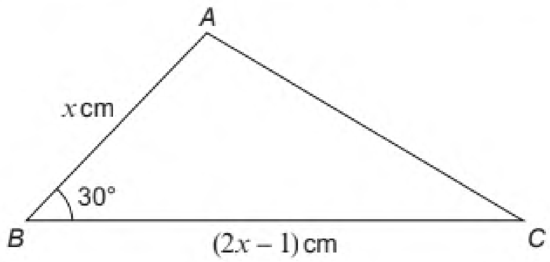
Use of a correct method to find the area of the triangle M1

Use of correct limits and trying to find the total area by adding the area of the triangle to the area under the curve m1

Shaded area = 64 + 52/3 = 244/3 (c.a.o.) A1

17.

(a) The diagram below shows a sketch of the triangle ABC with $AB = x$ cm, $BC = (2x - 1)$ cm and $A\hat{B}C = 30°$. The area of triangle ABC is $11{\cdot}25$ cm^2.



(i) Write down and simplify a quadratic equation satisfied by x. Hence show that $x = 5$.

(ii) Find the length of AC. Give your answer correct to one decimal place. [6]

(a) (i) $\frac{1}{2} \times x \times (2x - 1) \times \sin 30° = 11{\cdot}25$

(substituting the correct values and expressions in the correct places in the area formula) M1

$2x^2 - x - 45 = 0$ A1

An attempt to solve quadratic equation in x, either by using the quadratic formula or by getting the expression into the form $(ax + b)(cx + d)$, with $a \times c$ = candidate's coefficient of x^2 and $b \times d$ = candidate's constant m1

$(2x + 9)(x - 5) = 0 \Rightarrow x = 5$ (convincing) A1

(ii) $AC^2 = 5^2 + 9^2 - 2 \times 5 \times 9 \times \cos 30°$

(correct use of cos rule) M1

$AC = 5{\cdot}3$ cm (c.a.o.) A1