TRANSFORMATIONS OF FUNCTIONS

AS Unit 1: Pure Mathematics A

WJEC past paper questions: 2010 – 2017

Total marks available 72 (approximately 1 hour 20 minutes)

1. Figure 1 shows a sketch of the graph $y = f(x)$. The graph has a maximum point at (2, 5) and intersects the x-axis at the points (-2, 0) and (6, 0).

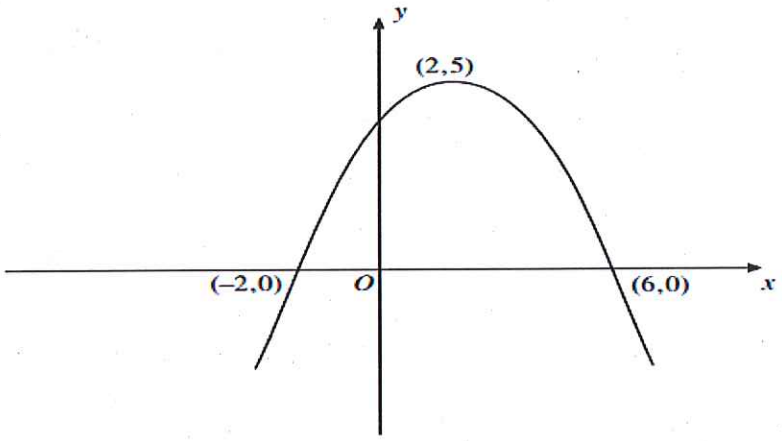


Figure 1

a) Sketch the graph of $y = f\left(\frac{x}{2}\right)$, indicating the coordinates of the stationary point and the coordinates of the points of intersection of the graph with the x-axis. (3)

b) Figure 2 shows a sketch of the graph having **one** of the following equations with an appropriate value of either p, q or r.

$y = f(x + p)$, where p is a constant
$y = f(x) + q$, where q is a constant
$y = rf(x)$, where r is a constant

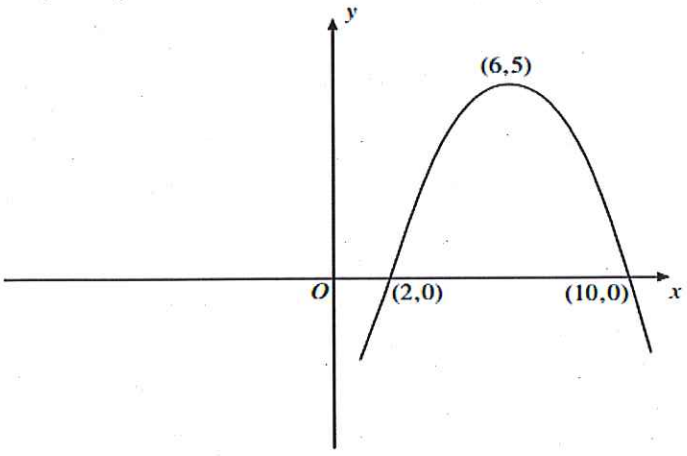
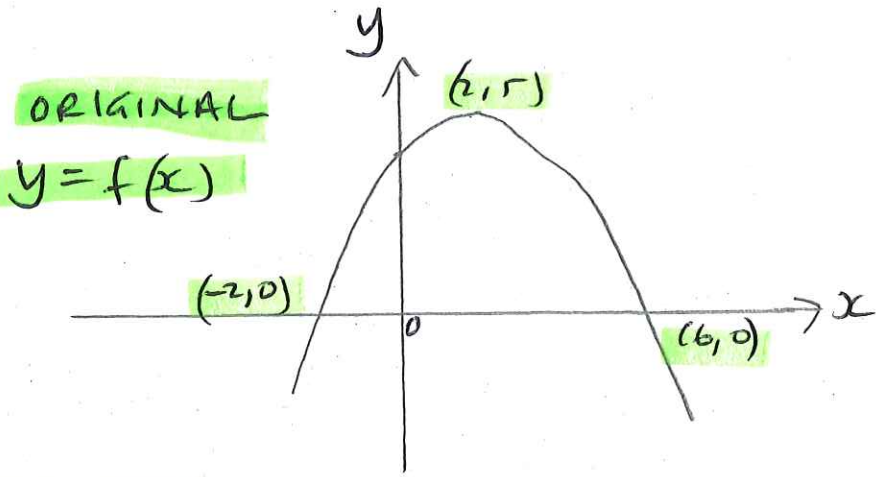


Figure 2

Write down the equation of the graph sketched in Figure 2, together with the value of the corresponding constant. (2)

(January 10)

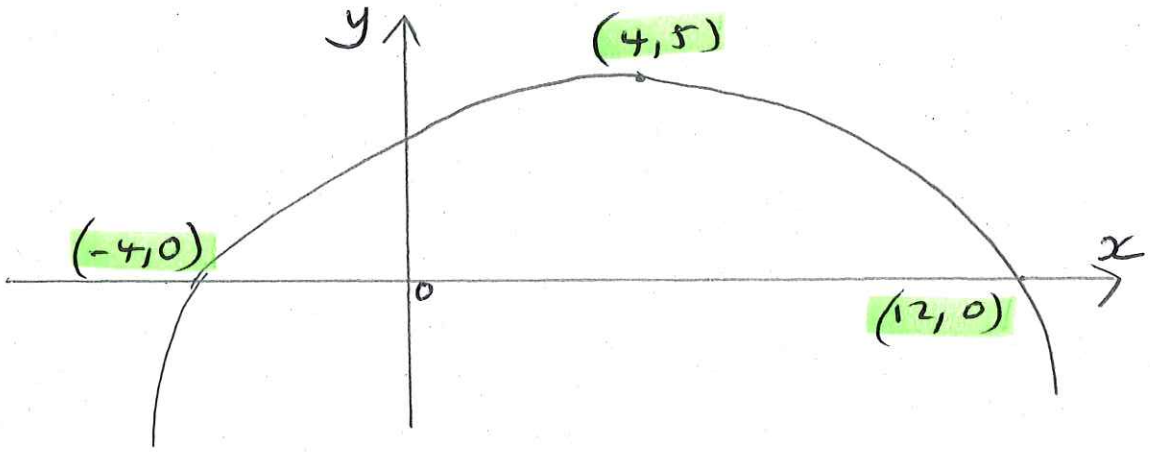
1a)



$y = f\left(\frac{x}{2}\right)$ ⇒ stretch along x-axis with scale factor 2

y coords stay the same

x coord × 2



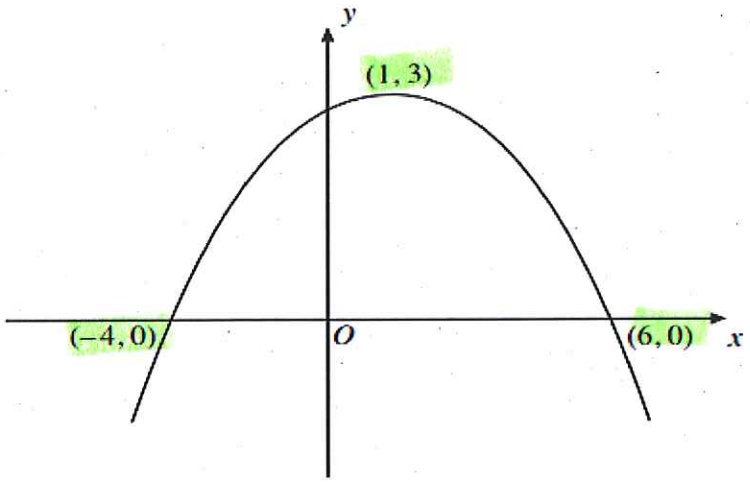
b) In figure 2, transformation is of the type

$$y = f(x+p)$$

because the graph is a translation along the x-axis. It moves 4 to the right so $p = -4$

$$y = f(x-4)$$

2. The diagram shows a sketch of the graph $y = f(x)$. The graph passes through the points (-4, 0) and (6, 0) and has a maximum point at (1, 3).



Sketch the following graphs, using a separate set of axes for each graph. In each case, you shouldindicate the coordinates of the stationary point and the coordinates of the points of intersection of the graph with the x-axis.

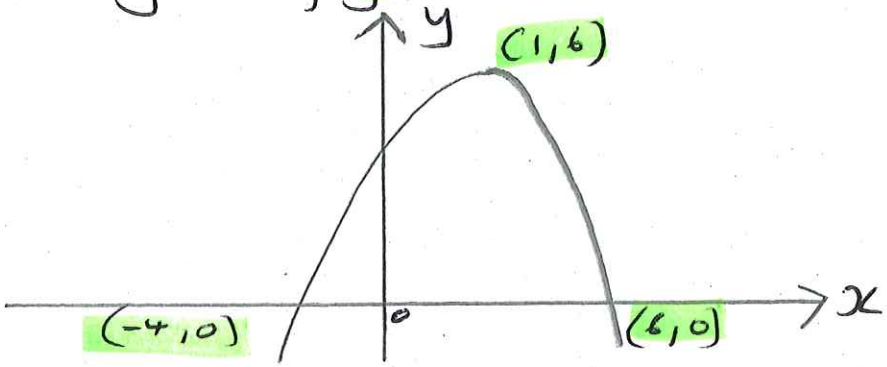
a) $y = 2f(x)$ (3)

b) $y = f(-x)$ (3)

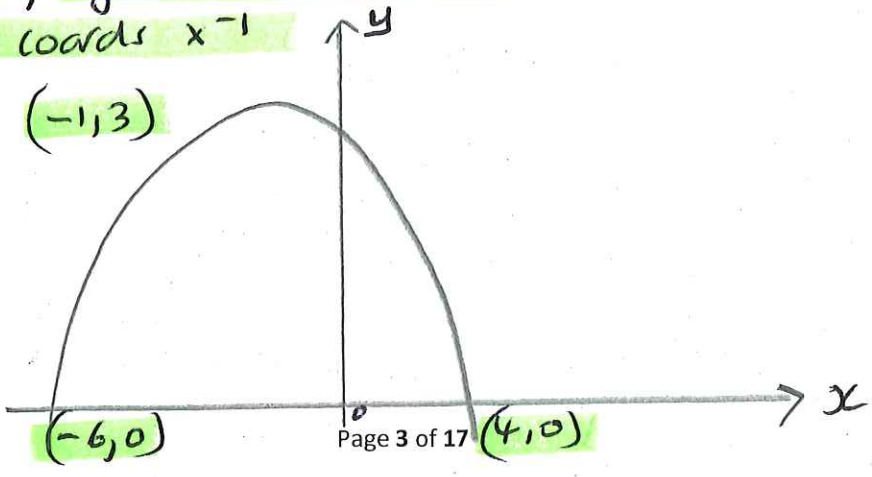
(Summer 10)

2a) $y = 2f(x)$, stretch along y axis with scale factor 2

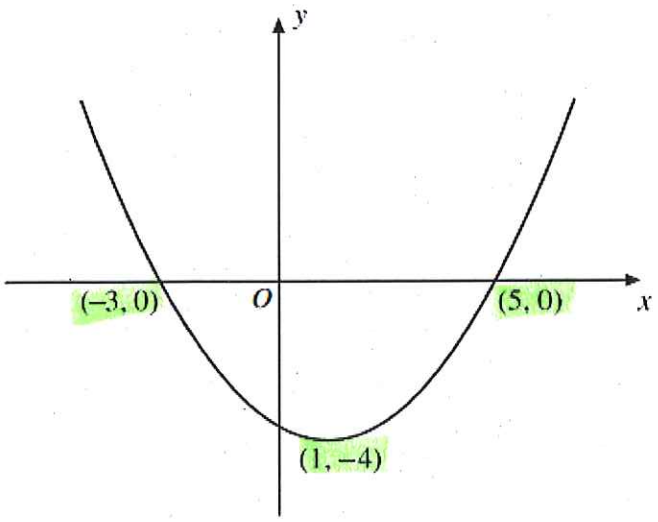
x coords stay same, y coords ×2



b) $y = f(-x)$, reflection across y axis, y coords stay the same, x coords x^{-1}



3. The diagram shows a sketch of the graph $y = f(x)$. The graph passes through the points (-3, 0) and (5, 0) and has a minimum point at (1, -4).



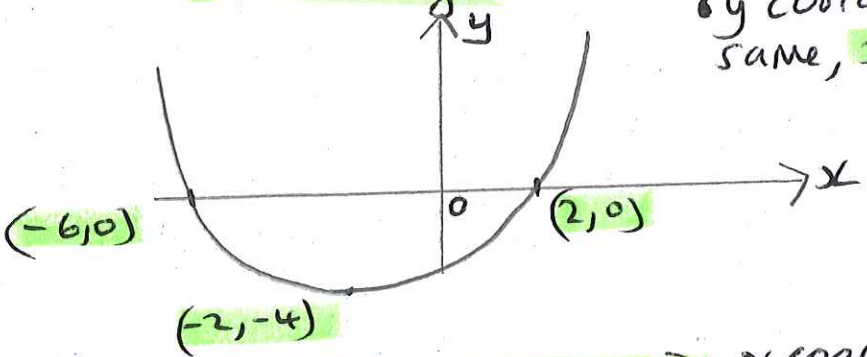
Sketch the following graphs, using a separate set of axes for each graph. In each case, you shouldindicate the coordinates of the stationary point and the coordinates of the points of intersection of the graph with the x-axis.

a) $y = f(x + 3)$ (3)

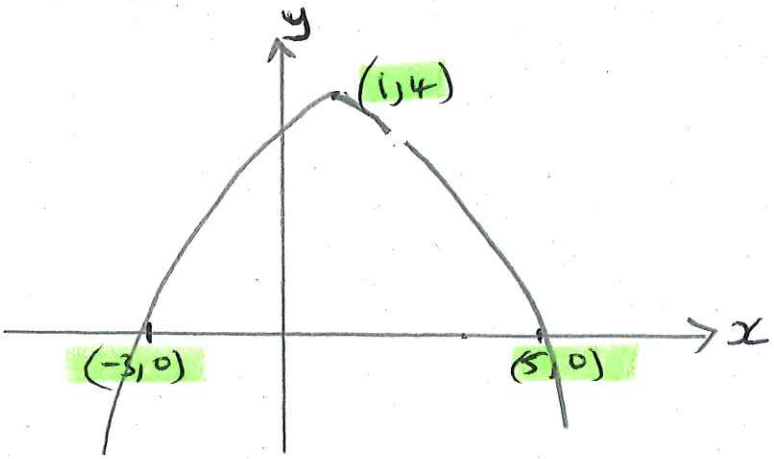
b) $y = -f(x)$ (3)

(January 11)

3a) $y = f(x+3)$, translation along x axis, 3 to the left (y coords stay the same, x coords -3)



b) $y = -f(x)$, reflection across x axis, x coords stay the same, y coords x -1



4. Figure 1 shows a sketch of the graph $y = f(x)$. The graph has a minimum point at (-3, -4) and intersects the x-axis at the points (-8, 0) and (2, 0).

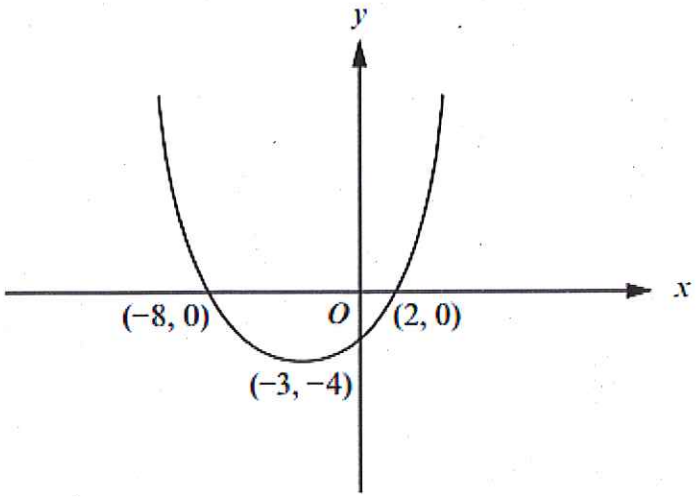


Figure 1

a) Sketch the graph of $y = f(x + 3)$, indicating the coordinates of the stationary point and the coordinates of the points of intersection of the graph with the x-axis. (3)

b) Figure 2 shows a sketch of the graph having **one** of the following equations with an appropriate value of either p, q or r.

$y = f(px)$, where p is a constant
$y = f(x) + q$, where q is a constant
$y = rf(x)$, where r is a constant

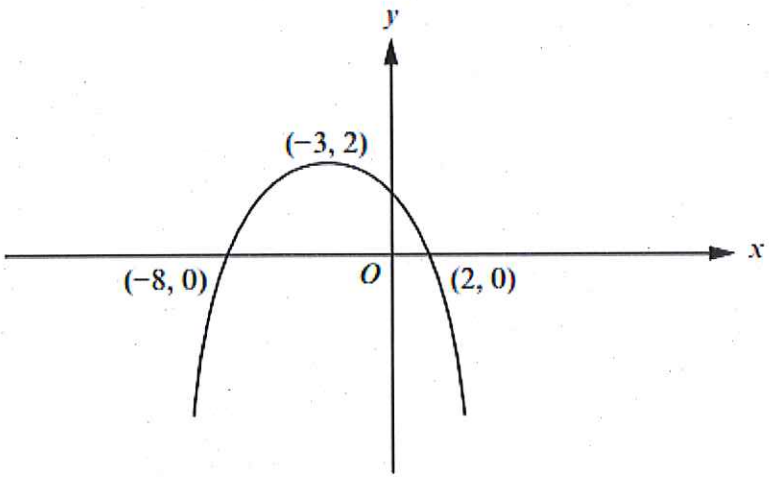


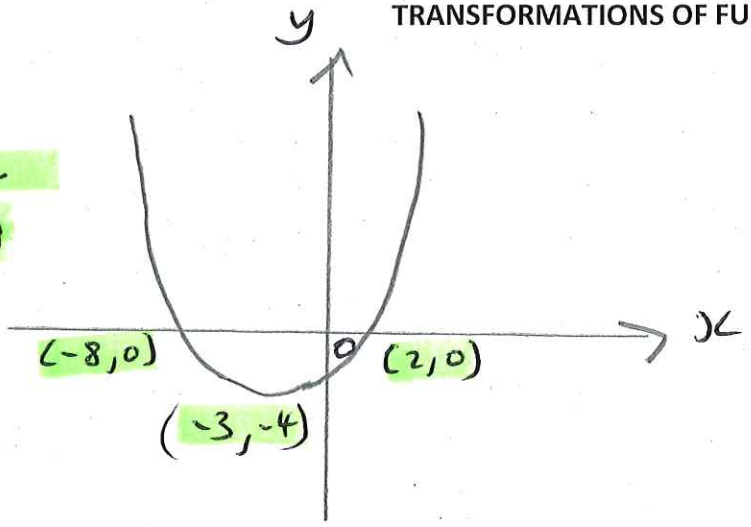
Figure 2

Write down the equation of the graph sketched in Figure 2, together with the value of the corresponding constant. (2)

(Summer 11)

4a)

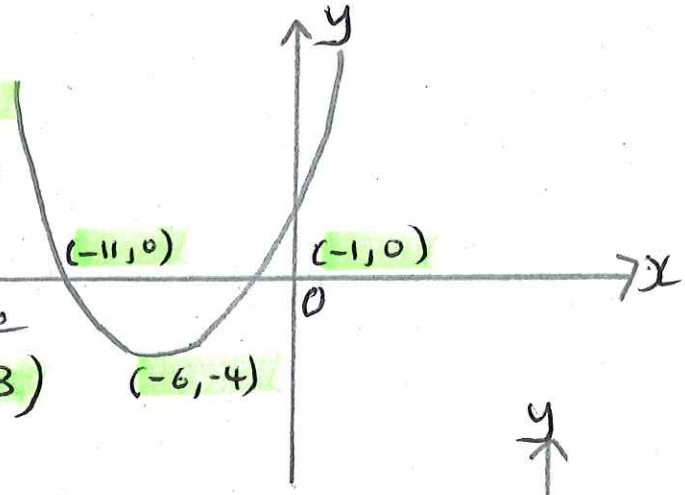
ORIGINAL
$y = f(x)$



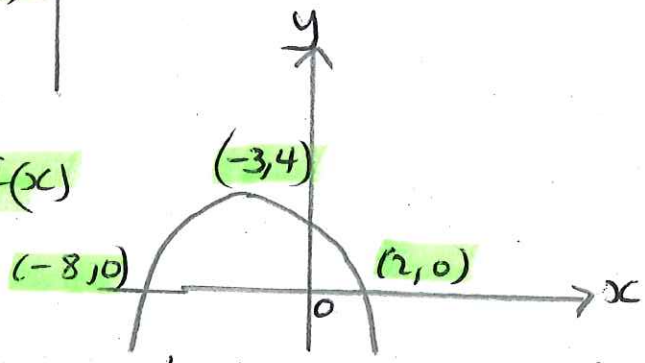
a) $y = f(x+3)$

TRANSLATION
along x-axis
3 to the left

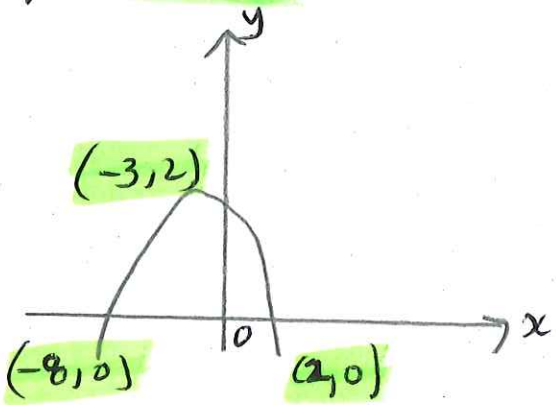
(y coords stay the same, x coords -3)



b) Firstly, think of $y = -f(x)$



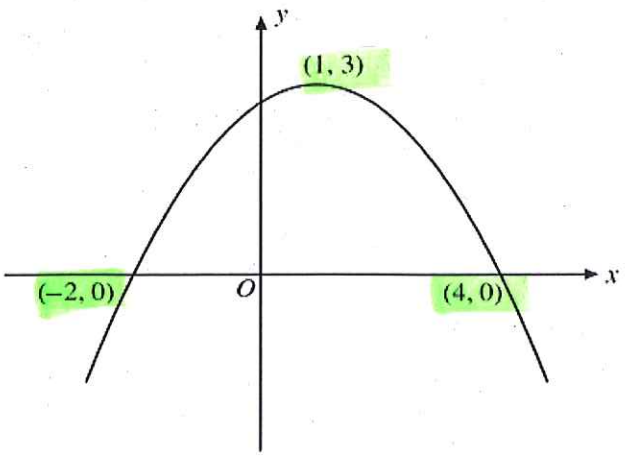
so it is some sort of reflection, but we want it 'squashed' $\times \frac{1}{2}$



$y = -\frac{1}{2} f(x)$

$(y = r f(x)$ where $r = -\frac{1}{2})$

5. The diagram shows a sketch of the graph $y = f(x)$. The graph has a maximum point at (1, 3) and intersects the x-axis at the points (-2, 0) and (4, 0).



a) Sketch the graph of $y = f(2x)$, indicating the coordinates of the stationary point and the coordinates of the points of intersection of the graph with the x-axis. (3)

b) i) Sketch the graph of $y = f(x) - 5$, indicating the coordinates of the stationary point.

ii) Given that f is a quadratic function, use the graph you have drawn in part i) to write down the number of real roots of the equation

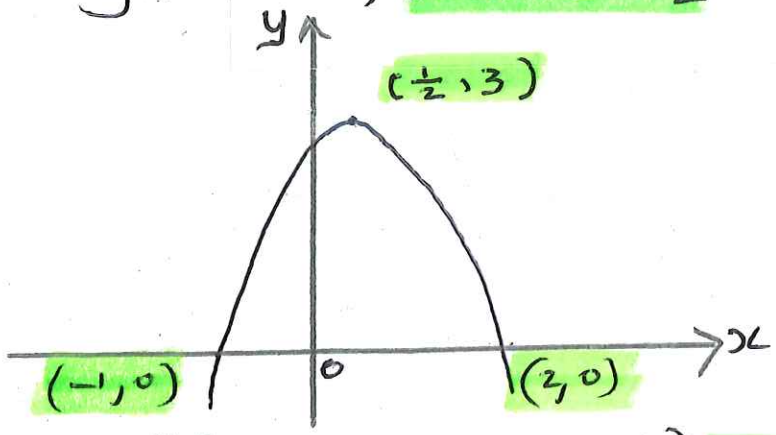
$$f(x) - 5 = 0.$$ (3)

(January 12)

5a) $y = f(2x)$

stretch, along x-axis, scale factor $\frac{1}{2}$

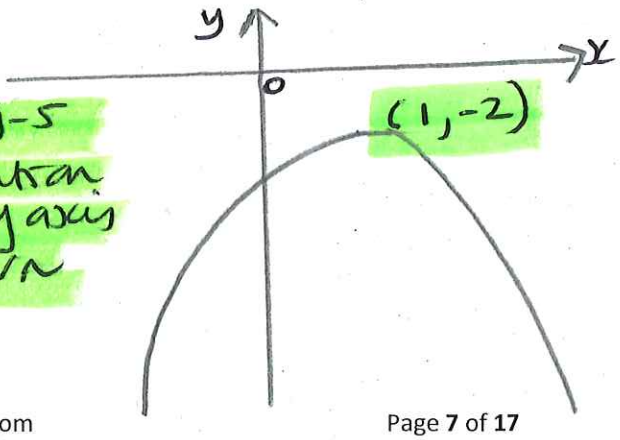
y coords stay the same, x-coords $\times \frac{1}{2}$



b) i)

$y = f(x) - 5$

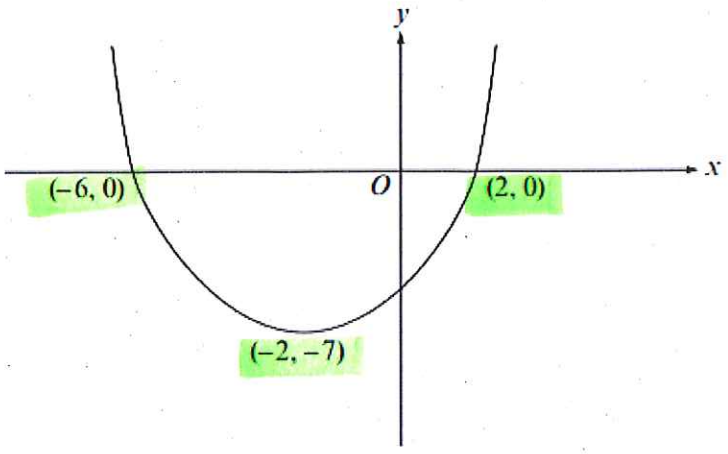
Translation along y axis 5 down



ii) zero

(doesn't cross the x axis at all)

6. The diagram shows a sketch of the graph $y = f(x)$. The graph passes through the points (-6, 0) and (2, 0) and has a minimum point at (-2, -7).



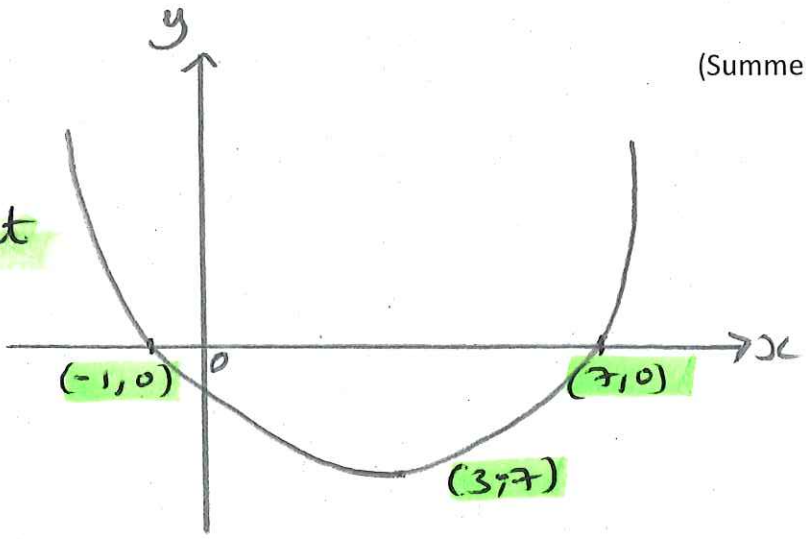
Sketch the following graphs, using a separate set of axes for each graph. In each case, you shouldindicate the coordinates of the stationary point and the coordinates of the points of intersection of the graph with the x-axis.

a) $y = f(x - 5)$ (3)

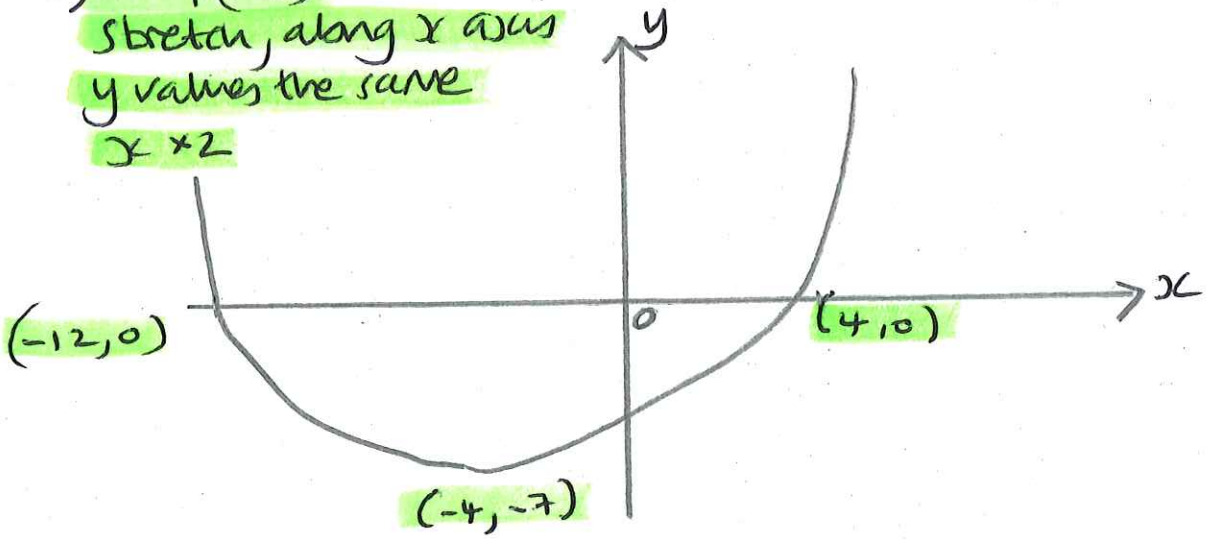
b) $y = f\left(\frac{x}{2}\right)$ (3)

(Summer 12)

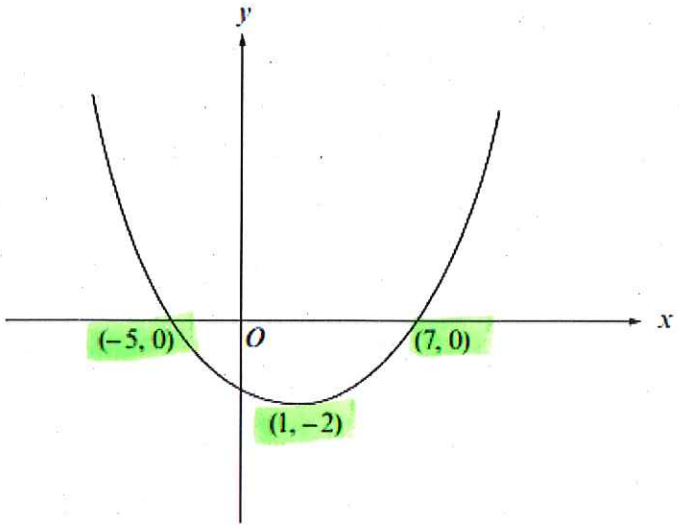
a) $y = f(x-5)$
Translation,
x direction
5 to the right



b) $y = f\left(\frac{x}{2}\right)$
Stretch, along x axis
y values the same
$x \times 2$



7. The diagram shows a sketch of the graph $y = f(x)$. The graph passes through the points (-5, 0) and (7, 0) and has a minimum point at (1, -2).



Sketch the following graphs, using a separate set of axes for each graph. In each case, you shouldindicate the coordinates of the stationary point and the coordinates of the points of intersection of the graph with the x-axis.

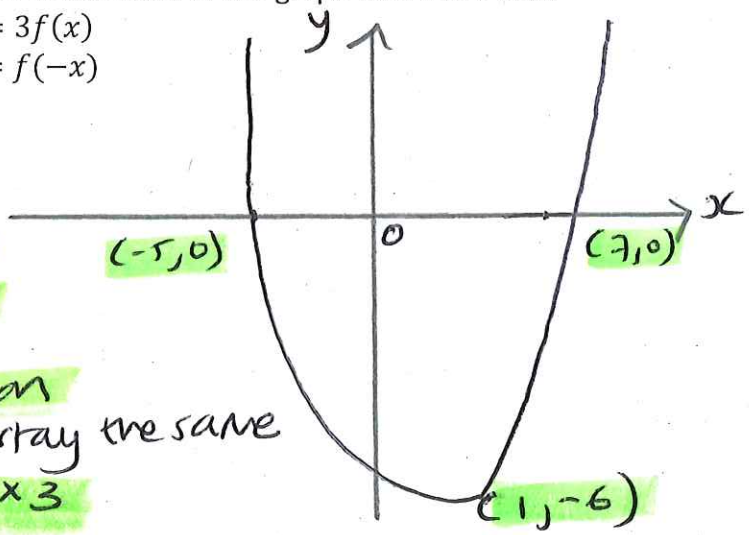
a) $y = 3f(x)$ (3)

b) $y = f(-x)$ (3)

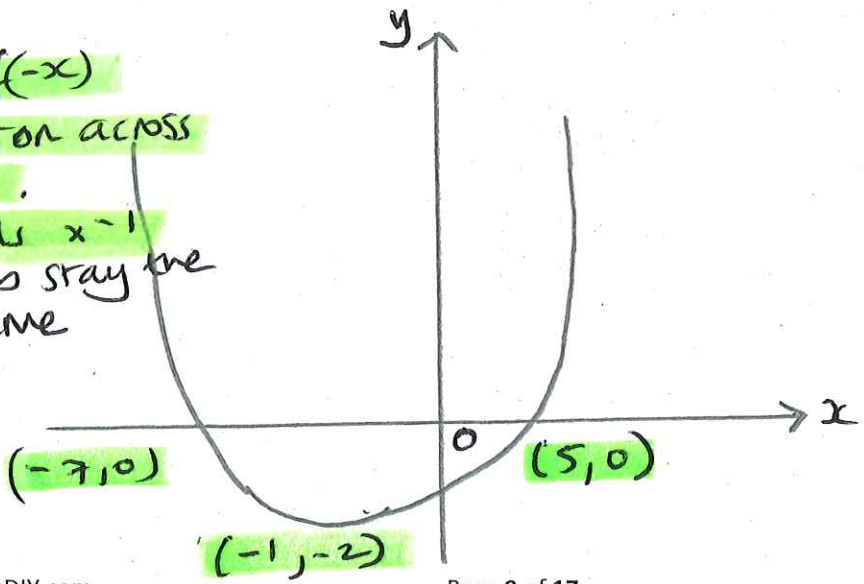
(January 13)

7a)

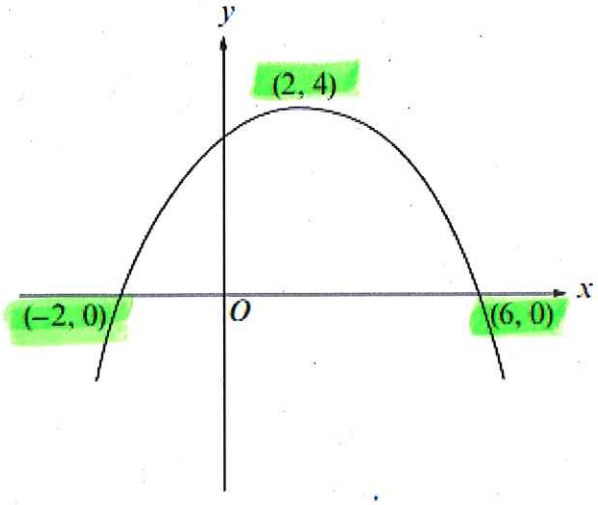
$y = 3f(x)$
stretch
x3
y direction
x coords stay the same
y coords x3



b) $y = f(-x)$
reflection across
y axis.
x coords x-1
y coords stay the same



8. The diagram shows a sketch of the graph $y = f(x)$. The graph passes through the points (-2, 0) and (6, 0) and has a maximum point at (2, 4).



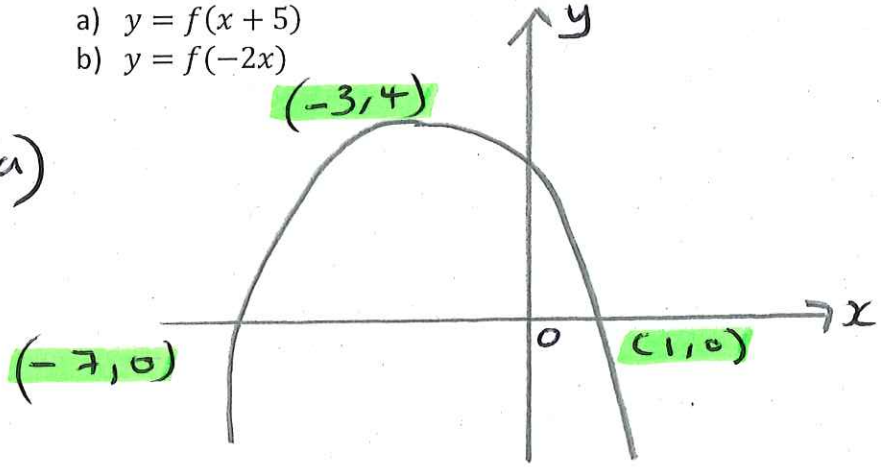
Sketch the following graphs, using a separate set of axes for each graph. In each case, you shouldindicate the coordinates of the stationary point and the coordinates of the points of intersection of the graph with the x-axis.

a) $y = f(x + 5)$ (3)

b) $y = f(-2x)$ (3)

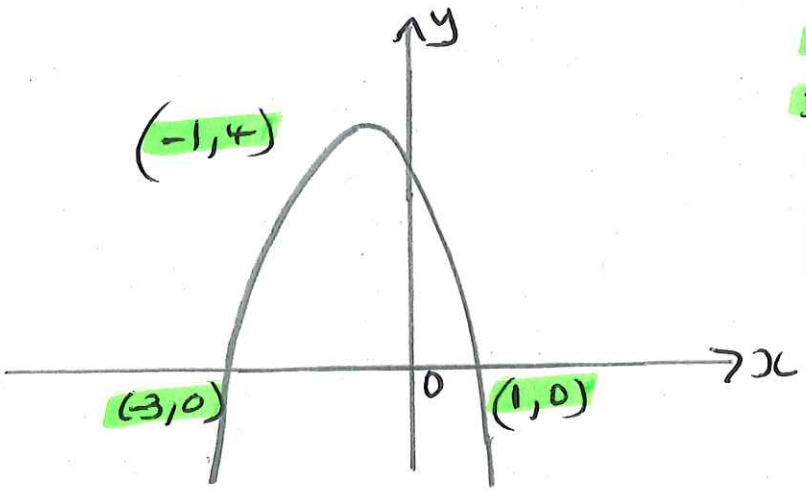
(Summer 13)

8a)



$y = f(x+5)$
Translation along x axis
y coords stay the same
x coords subtract 5

b)



$y = f(-2x)$, reflect & stretch, x direction
y coords stay same
x coords $x - \frac{1}{2}$

9. **Figure 1** shows a sketch of the graph $y = f(x)$. The graph has a maximum point at (2, 6) and intersects the x-axis at the points (-4, 0) and (8, 0).

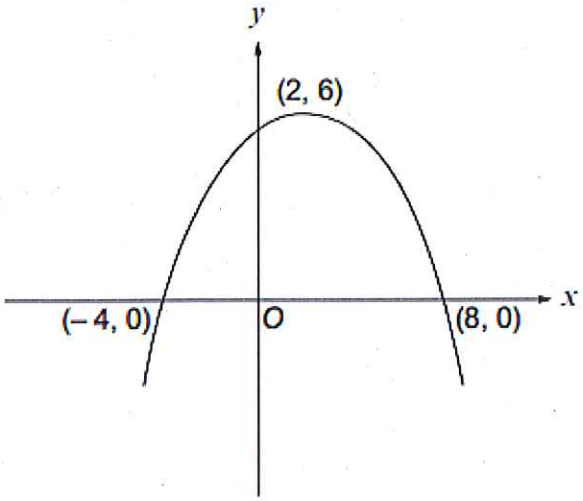


Figure 1

a) Sketch the graph of $y = f(x - 3)$, indicating the coordinates of the stationary point and the coordinates of the points of intersection of the graph with the x-axis. (3)

b) **Figure 2** shows a sketch of the graph having **one** of the following equations with an appropriate value of either p, q or r.

$y = f(x) + p$, where p is a constant
$y = f(qx)$, where q is a constant
$y = rf(x)$, where r is a constant

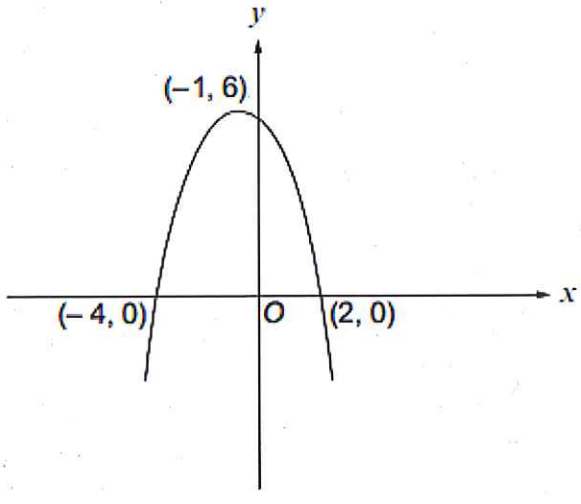
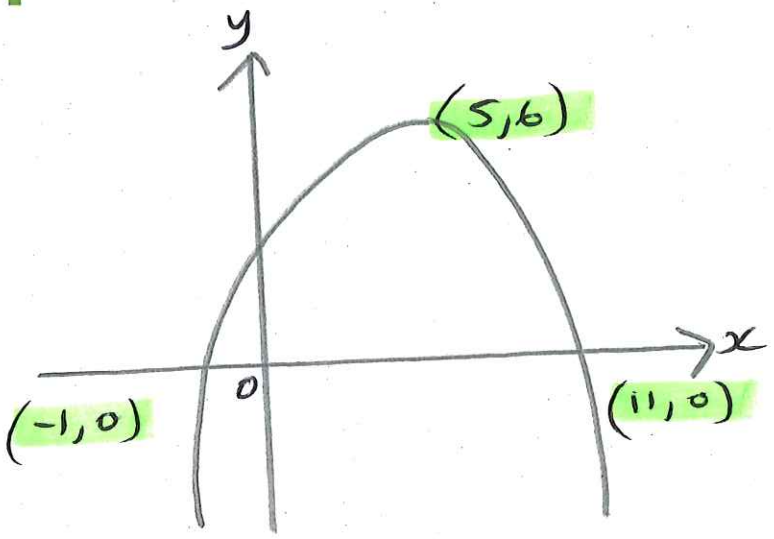


Figure 2

Write down the equation of the graph sketched in **Figure 2**, together with the value of the corresponding constant. (2)

(January 14)

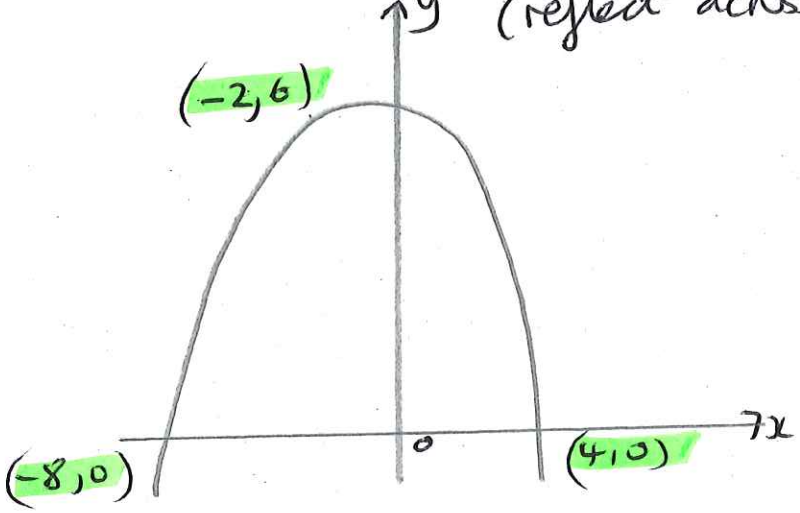
9a)



$y = f(x-3)$

Translation, x direction 3 to the right (y coords stay the same, x coords add on 3)

b) To help, first think of $y = f(-x)$, would be (reflect across y axis, x coords x -1)



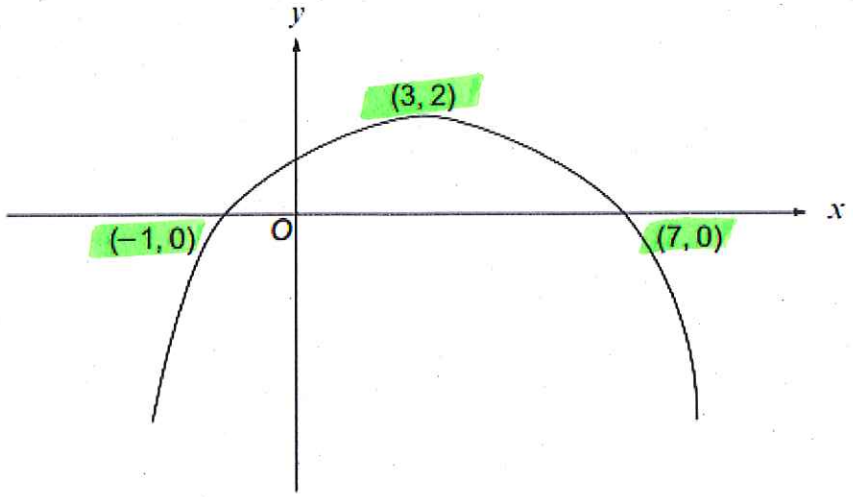
but we actually require the x coordinates to be half of this

therefore $y = f(-2x)$

The equation of the graph is of the type

$y = f(qx)$ where $q = -2$

10. The diagram shows a sketch of the graph $y = f(x)$. The graph passes through the points (-1, 0) and (7, 0) and has a maximum point at (3, 2).

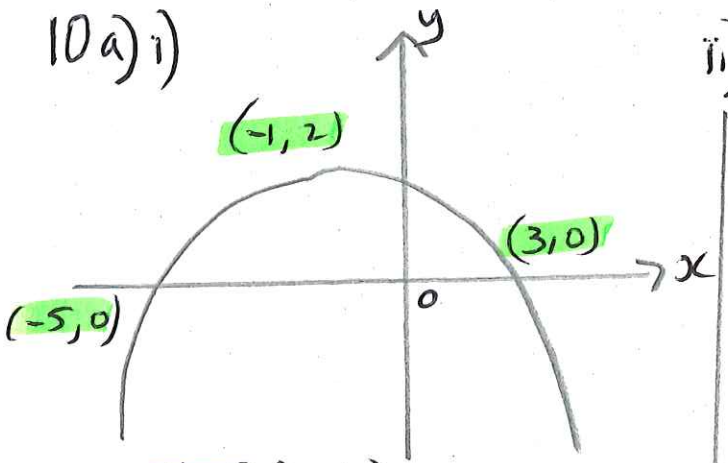


a) Sketch the following graphs, using a separate set of axes for each graph. In each case, you should indicate the coordinates of the stationary point and the coordinates of the points of intersection of the graph with the x-axis.
   i) $y = f(x + 4)$
   ii) $y = -2f(x)$ (6)

b) Hence write down one root of the equation
   $f(x + 4) = -2f(x) + 4$. (1)

(Summer 14)

10 a) i)



$y = f(x+4)$

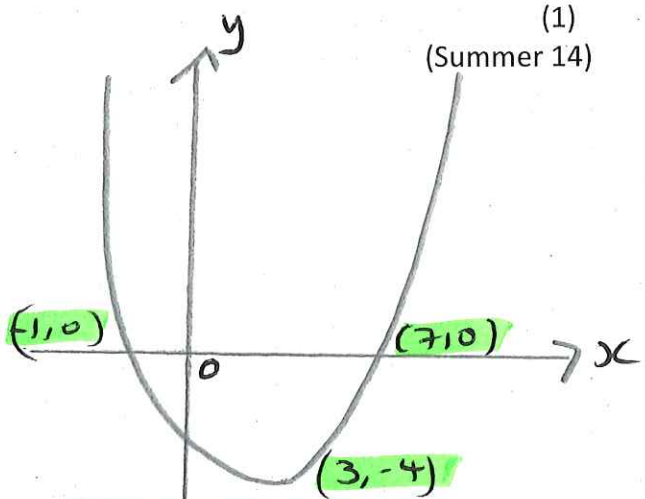
Translation, x direction

4 to the left

x coords subtract 4

y coords stay the same

ii)



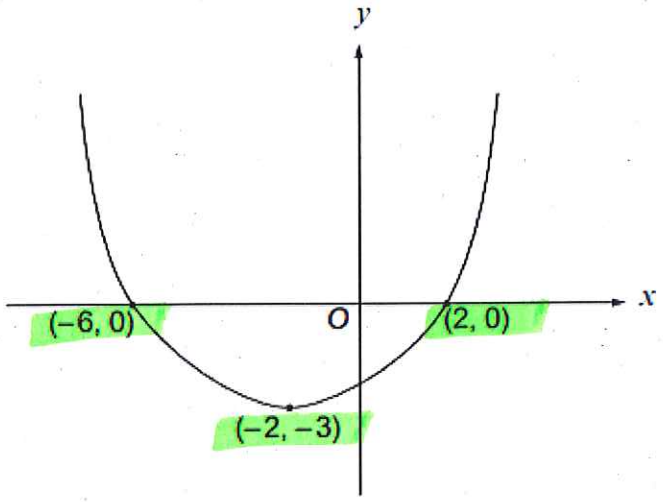
$y = -2f(x)$

you can think of this in 2 parts

If it helps, first reflect across x axis, the x2 all y coordinates

b) $x = 3$

11. The diagram shows a sketch of the graph $y = f(x)$. The graph passes through the points (-6, 0) and (2, 0) and has a minimum point at (-2, -3).

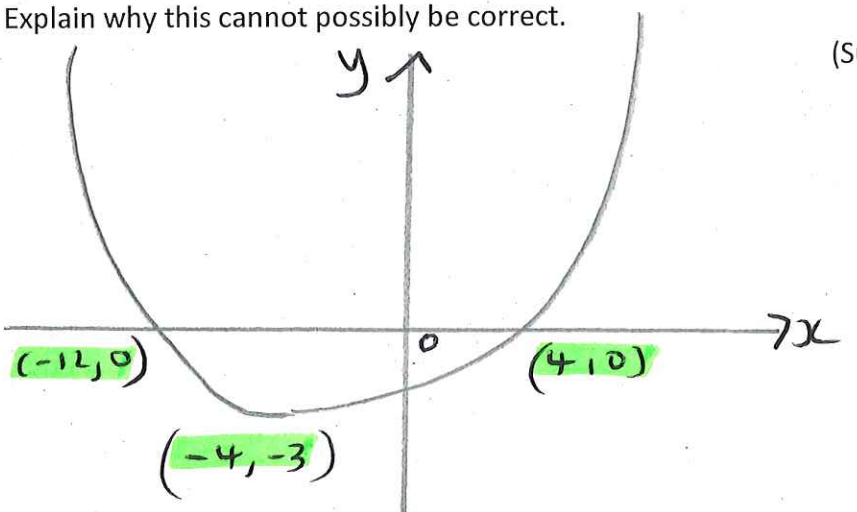


a) Sketch the graph of $y = f\left(\frac{1}{2}x\right)$, indicating the coordinates of the stationary point and the coordinates of the points of intersection of the graph with the x-axis. (3)

b) Angharad is asked by her teacher to draw the graph of $y = af(x)$ for various non-zero values of the constant a. One of Angharad's graphs passes through the origin O. Explain why this cannot possibly be correct. (1)

(Summer 15)

a)



$y = f\left(\frac{1}{2}x\right)$, stretch x 2 in the x direction

x coordinates x 2, y coordinates stay the same

b) $y = af(x)$, means a stretch 'x a' in y direction any graph of this form will always intersect with the x-axis at (-6, 0) and (2, 0). Therefore it will never pass through the origin.

12. Figure 1 shows a sketch of the graph $y = f(x)$. The graph has a minimum point at (1, -3) and intersects the x-axis at the points (-4, 0) and (6, 0).

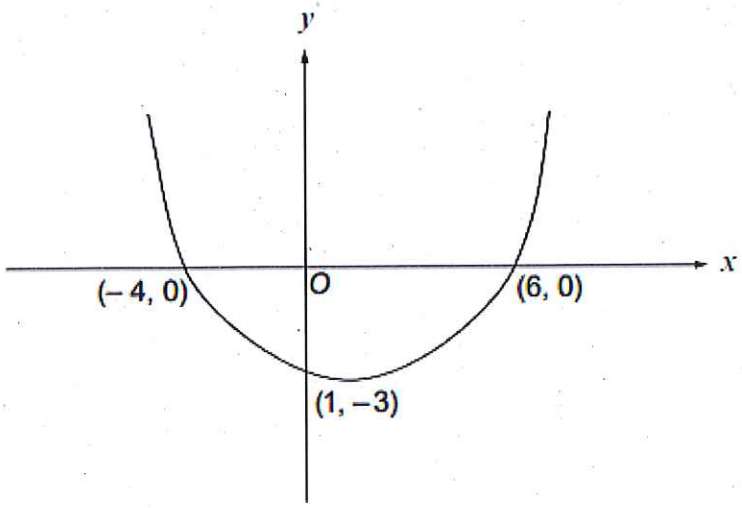


Figure 1

a) Sketch the graph of $y = -3f(x)$, indicating the coordinates of the stationary point and the coordinates of the points of intersection of the graph with the x-axis. (3)

b) **Figure 2** shows a sketch of the graph $y = g(x)$, where

$g(x) = f(x) + p$, where p is a constant,
or $g(x) = f(qx)$, where q is a constant,
or $g(x) = rf(x)$, where r is a constant,
or $g(x) = f(x + s)$, where s is a constant.

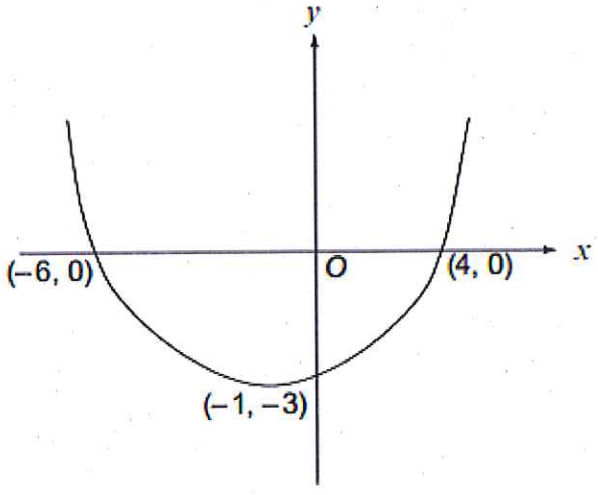
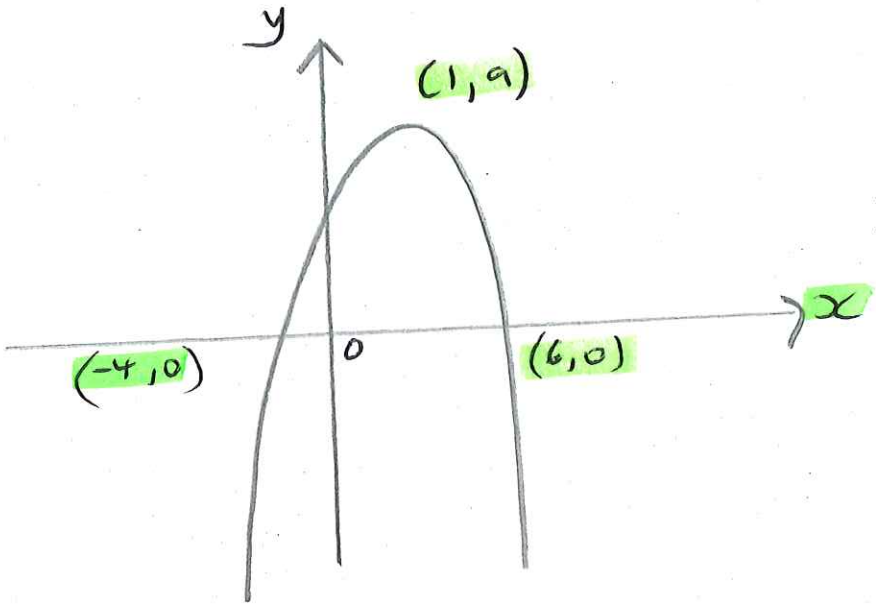


Figure 2

The function g can in fact be any one of **two** of the above functions. In each of these two cases, write down the expression for $g(x)$, including the value of the corresponding constant. (2)

(Summer 16)

12a)



$y = -3f(x)$

It might help to think of this graph in two parts.

First, $y = -f(x)$ means reflect the original $f(x)$ across the x axis. x coords stay the same, y coords 'x' -1

Then, do the x3 stretch bit, x coords stay the same y coords x3

b) Translation, x direction, 2 to the left

so,

$g(x) = f(x+s)$ where $s = 2$

$$g(x) = f(x+2)$$

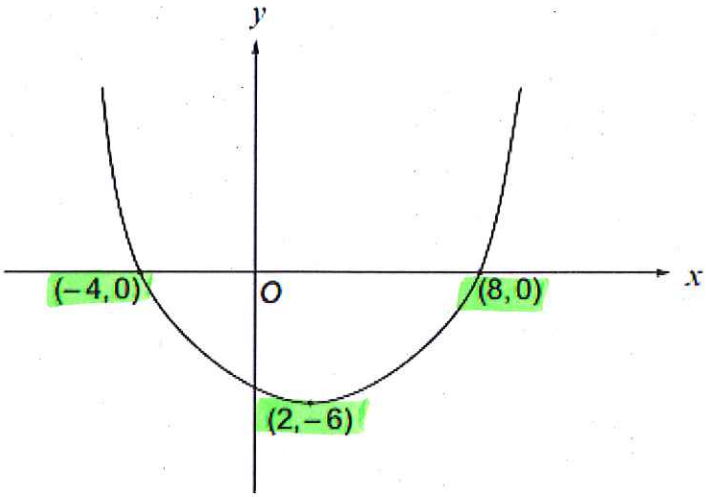
or

Reflection across the y axis. All the y coordinates stay the same, the x coordinates 'x' -1

so, $g(x) = f(qx)$ where $q = -1$

$$g(x) = f(-x)$$

13. The diagram shows a sketch of the graph $y = f(x)$. The graph passes through the points (-4, 0) and (8, 0) and has a minimum point at (2, -6).
hyguukh

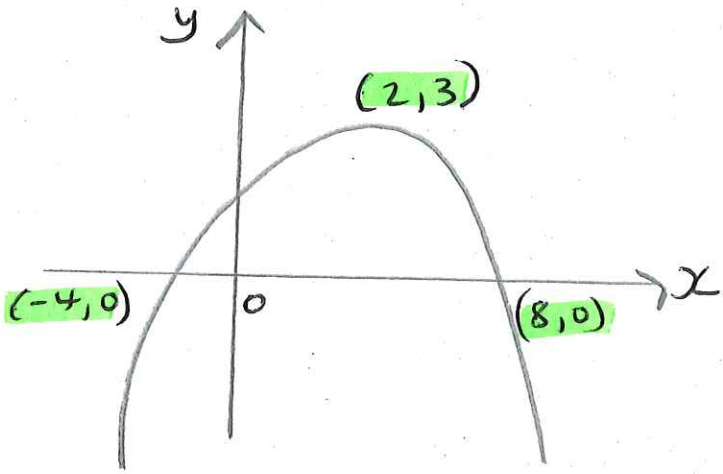


a) Sketch the graph of $y = -\frac{1}{2}f(x)$, indicating the coordinates of the stationary point and the coordinates of the points of intersection of the graph with the x-axis. (3)

c) Siân is asked by her teacher to draw the graph of $y = f(ax)$ for various non-zero values of the constant a. Write down two facts about the stationary point on Siân's graph which will always be true whatever her choice of a. (2)

(Summer 17)

13 a)



$y = -\frac{1}{2}f(x)$

first think of
$y = -f(x)$
reflect across x axis
y coordinates 'x' -1
Then ($\frac{1}{2}$) bit means
stretch $\times \frac{1}{2}$

b) The stationary point will always be a MINIMUM.
The y-coordinate will always be -6.