QUADRATICS & INEQUALITIES

AS Unit 1: Pure Mathematics A

WJEC past paper questions: 2010 – 2017

Total marks available 89 (approximately 1 hour 50 minutes)

1. a) Find the range of values of k for which the quadratic equation
$$kx^2 + 3x - 5 = 0$$
has no real roots. (4)

b) Solve the inequality $2x^2 - x - 6 > 0$. (3)

(January 10)

1a) $kx^2+3x-5=0$

No real roots means $b^2-4ac<0$

$a=k$, $b=3$, $c=-5$

$3^2-4(k)(-5)<0$

$9+20k<0$

$k<-\frac{9}{20}$

b) $2x^2-x-6>0$

$a=2$, $b=-1$, $c=-6$

'x' ac	4 'b
-12	-1

$-4, 3$

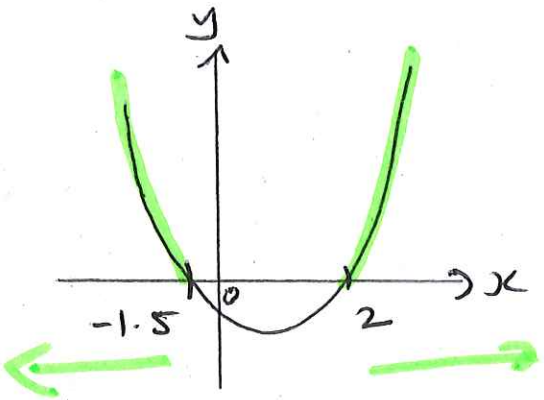
$2x^2-4x+3x-6>0$

$2x(x-2)+3(x-2)>0$

$(x-2)(2x+3)>0$

If you drew $y=(x-2)(2x+3)$ where does it cut the x axis?

$x=2$ and $x=-\frac{3}{2}$



we want $(x-2)(2x+3)>0$

$x<-\frac{3}{2}$ or $x>2$

2. a) Find the range of values of k for which the quadratic equation

$$2x^2 + kx + 18 = 0$$

has no real roots. (4)

b) Solve the inequality $10x^2 - x - 3 \geq 0$. (3)

(Summer 10)

2a) $2x^2+kx+18=0$

$a=2$ $b=k$ $c=18$

No real roots means $b^2-4ac<0$

$k^2-4(2)(18)<0$

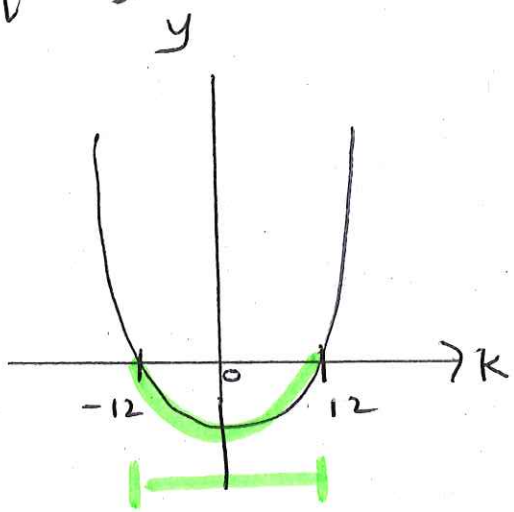
$k^2-144<0$ (difference of 2 squares)

$(k-12)(k+12)<0$

If you drew $y=(k-12)(k+12)$

where does it cut the k axis?

$k=12$ and $k=-12$



we want

$(k-12)(k+12)<0$

$-12<k<12$

b) $10x^2-x-3\geq 0$

$a=10$ $b=-1$ $c=-3$

(×) ac	(+) b
-30	-1

$-6, 5$

$10x^2+5x-6x-3\geq 0$

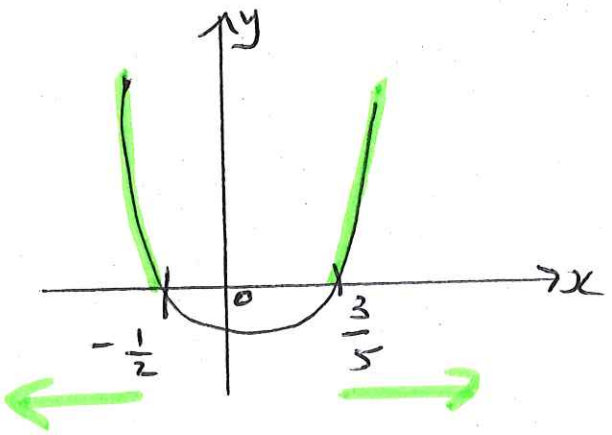
$5x(2x+1)-3(2x+1)\geq 0$

$(2x+1)(5x-3)\geq 0$

If you drew $y=(2x+1)(5x-3)$

where does it cut the x axis

$x=-\frac{1}{2}$ $x=\frac{3}{5}$



$x\leq -\frac{1}{2}$ or $x\geq \frac{3}{5}$

3. Given that the quadratic equation

$$2x^2 + (3k-1)x + (3k^2-1) = 0$$

has two distinct real roots, show that

$$5k^2 + 2k - 3 < 0.$$

Find the range of values of k satisfying this inequality. (7)

(January 11)

3. $2x^2 + (3k-1)x + (3k^2-1) = 0$

has two distinct roots

$\Rightarrow b^2 - 4ac > 0$

$a = 2$

$b = 3k-1$

$c = 3k^2-1$

$(3k-1)^2 - 4(2)(3k^2-1) > 0$

$9k^2 - 6k + 1 - 24k^2 + 8 > 0$

$-15k^2 - 6k + 9 > 0 \quad (\div -3)$

$5k^2 + 2k - 3 < 0$

(careful with inequality sign here)

$a = 5 \quad b = 2 \quad c = -3$

'×' ac	'+' b
−15	2

$-3, 5$

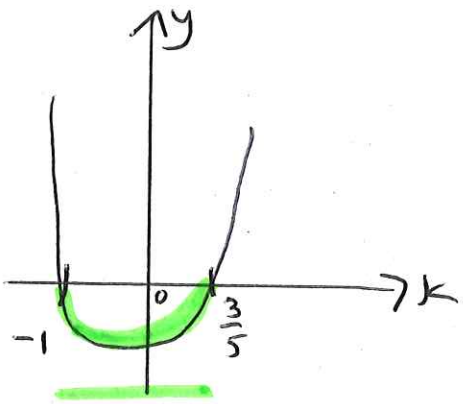
$5k^2 + 5k - 3k - 3 < 0$

$5k(k+1) - 3(k+1) < 0$

$(k+1)(5k-3) < 0$

If you drew $y = (k+1)(5k-3)$ where would it cut the k axis?

$k = -1$ and $k = \frac{3}{5}$



we want $(k+1)(5k-3) < 0$

$$-1 < k < \frac{3}{5}$$

4. The curve C has equation

$$y = x^2 + (4k + 3)x + 7,$$

and the line L has equation

$$y = x + k,$$

where k is a constant.

Given that L and C intersect at two distinct points,

a) show that $4k^2 + 5k - 6 > 0$, (6)

b) find the range of values of k satisfying this inequality. (3)

(Summer 11)

4a) $y = x^2 + (4k+3)x + 7$

$y = x + k$

Solve simultaneously

$\therefore \quad x + k = x^2 + 4kx + 3x + 7$

$0 = x^2 + 4kx + 3x - x + 7 - k$

$0 = x^2 + (4k+2)x + (7-k)$

$a = 1$
$b = 4k+2$
$c = 7-k$

Given that they intersect at two distinct points.

This means $b^2 - 4ac > 0$

$(4k+2)^2 - 4(1)(7-k) > 0$

$16k^2 + 16k + 4 - 28 + 4k > 0$

$16k^2 + 20k - 24 > 0 \quad (\div 4)$

$4k^2 + 5k - 6 > 0$

$a = 4$
$b = 5$
$c = -6$

'x' ac	(+) b
-24	5

8, -3

$4k^2 + 8k - 3k - 6 > 0$

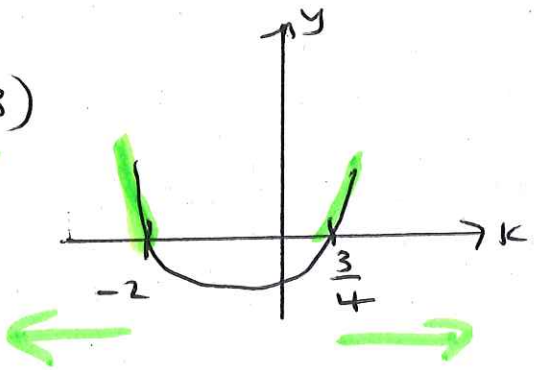
$4k(k+2) - 3(k+2) > 0$

$(k+2)(4k-3) > 0$

If you drew $y = (k+2)(4k-3)$

where does it cut the k-axis?

$k = -2, \quad k = \frac{3}{4}$



we want $(k+2)(4k-3) > 0$

$k < -2$ or $k > \frac{3}{4}$

5. Given that the quadratic equation

$$(k+6)x^2 + 4x + (k+3) = 0$$

has no real roots, show that

$$k^2 + 9k + 14 > 0.$$

Find the range of values of k satisfying this inequality. (7)

(January 12)

5. No real roots means $b^2 - 4ac < 0$

$a = k+6$

$b = 4$

$c = k+3$

$4^2 - 4(k+6)(k+3) < 0$

$4^2 - 4(k^2+9k+18) < 0$

$16 - 4k^2 - 36k - 72 < 0$

$-4k^2 - 36k - 56 < 0$

$k^2 + 9k + 14 > 0$ $(\div -4)$

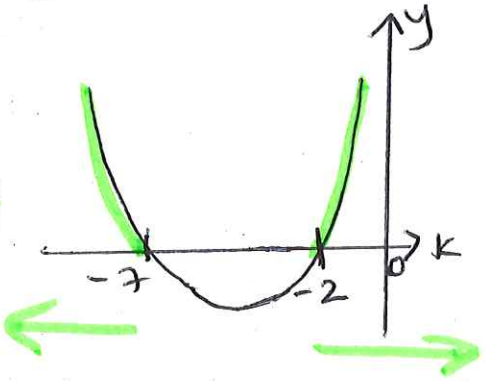
(careful with inequality sign here)

$(k+7)(k+2) > 0$

If we drew $y = (k+7)(k+2)$

where does it cut the k axis?

$k = -7$ and $k = -2$



we want $(k+7)(k+2) > 0$

$k < -7$ or $k > -2$

6. a) Show that the equation

$$x^2 + (2k-1)x + (k^2 - k + 2) = 0$$

has no real roots, whatever the value of the constant k. (4)

b) Find the range of values of x satisfying the inequality.

$$3x^2 + 16x - 12 > 0.$$ (3)

(Summer 12)

6a) $x^2 + (2k-1)x + (k^2 - k + 2) = 0$

No real roots?

Consider the discriminant

$b^2 - 4ac$

$a = 1$

$b = (2k-1)$

$c = (k^2 - k + 2)$

$= (2k-1)^2 - 4(1)(k^2 - k + 2)$

$= 4k^2 - 4k + 1 - 4k^2 + 4k - 8$

$= -7 < 0$ discriminant < 0, so no real roots

discriminant does not depend on k

no real roots whatever the value of k.

b) $3x^2 + 16x - 12 > 0$

$3x^2 + 18x - 2x - 12 > 0$

$3x(x+6) - 2(x+6) > 0$

$(3x-2)(x+6) > 0$

$a = 3$

$b = 16$

$c = -12$

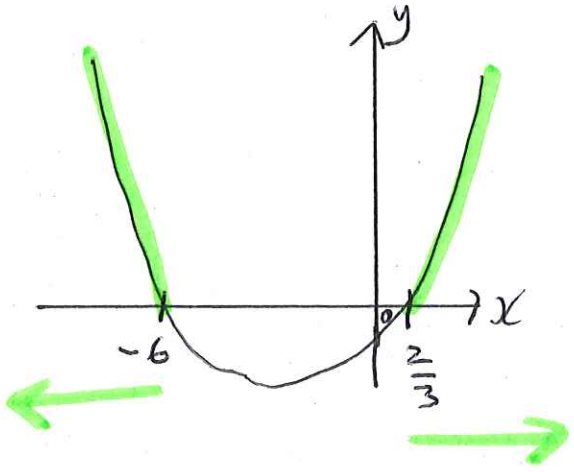
'+' b	'×' ac
16	-36

18, -2

If we drew $y = (3x-2)(x+6)$

Where does it cut the x axis?

$x = \frac{2}{3}$ and $x = -6$



We want

$(3x-2)(x+6) > 0$

$x < -6$ or $x > \frac{2}{3}$

7. a) Find the range of values of k for which the quadratic equation

$$5x^2 + 6x - 3k = 0$$

has two distinct real roots. (4)

b) Solve the inequality

$$2x^2 - 11x + 15 \leq 0.$$ (3)

(January 13)

7a) $5x^2 + 6x - 3k = 0$

two distinct real roots $\therefore$ $b^2 - 4ac > 0$

$a = 5$

$b = 6$

$c = -3k$

$6^2 - 4(5)(-3k) > 0$

$36 + 60k > 0$

$60k > -36$

$k > \frac{-36}{60}$, $k > -\frac{3}{5}$

b) $2x^2 - 11x + 15 \leq 0$

$a = 2 \quad b = -11 \quad c = 15$

'x'ac	'+'b
30	-11

$-6, -5$

$2x^2 - 6x - 5x + 15 \leq 0$

$2x(x-3) - 5(x-3) \leq 0$

$(x-3)(2x-5) \leq 0$

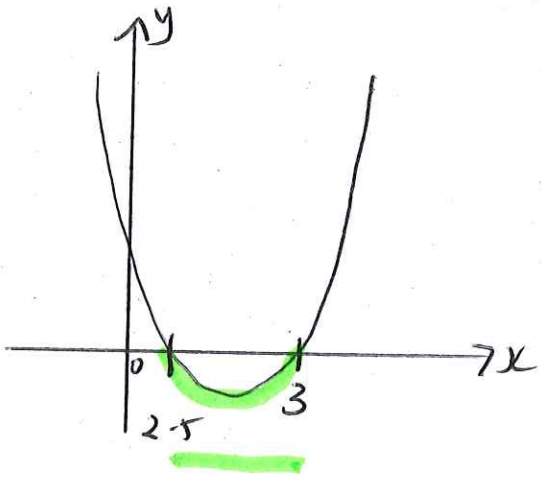
If we drew $y = (x-3)(2x-5)$

where would it cut the x axis?

$x = 3$ and $x = 2.5$

we want

$(x-3)(2x-5) \leq 0$



$2.5 \leq x \leq 3$

8. a) i) Assuming that the quadratic equation
$$(k+1)x^2 + (4k+1)x + (k-5) = 0$$
has **two equal** roots, show that $4k^2 + 8k + 7 = 0$
ii) Hence show that there are **no real** values of k such that the quadratic equation
$$(k+1)x^2 + (4k+1)x + (k-5) = 0$$
has two equal roots. (6)

b) Find the range of values of x satisfying the inequality.
$$4x^2 - 9x - 9 \geq 0.$$ (3)

(Summer 13)

8a)i) $(k+1)x^2 + (4k+1)x + (k-5) = 0$

given two equal roots

therefore, $b^2 - 4ac = 0$

$a = k+1$
$b = 4k+1$
$c = k-5$

$(4k+1)^2 - 4(k+1)(k-5) = 0$
$16k^2 + 8k + 1 - 4k^2 + 16k + 20 = 0$
$12k^2 + 24k + 21 = 0 \quad (\div 3)$
$4k^2 + 8k + 7 = 0$

ii) Consider the discriminant of $4k^2 + 8k + 7 = 0$

$a = 4$
$b = 8$
$c = 7$

$b^2 - 4ac = 64 - 4(4)(7)$
$= 64 - 112 = -56$

∴ because the discriminant is negative there are no real values of k such that the equation has equal roots.

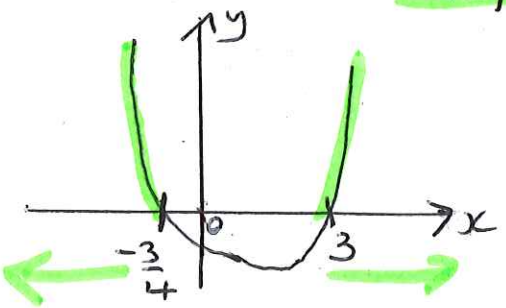
b) $4x^2 - 9x - 9 \geq 0$
$4x^2 - 12x + 3x - 9 \geq 0$
$4x(x-3) + 3(x-3) \geq 0$
$(x-3)(4x+3) \geq 0$

If drew $y = (x-3)(4x+3)$
crosses x axis at $x = 3$, $x = -\frac{3}{4}$

$a = 4$
$b = -9$
$c = -9$

'x' ac	(+) b
-36	-9

$-12, 3$



$x \leq -\frac{3}{4}$ or $x \geq 3$

9. Given that the quadratic equation

$$(2k-3)x^2+8x+(2k+3)=0$$

has no real roots, show that k satisfies an inequality of the form

$$m-nk^2<0,$$

where m, n are integers whose values are to be found.

Hence find the range of values of k such that the quadratic equation

$$(2k-3)x^2+8x+(2k+3)=0$$

has no real roots. (6)

(January 14)

9. If $(2k-3)x^2+8x+(2k+3)=0$

has no real roots

then $b^2-4ac<0$

$a=2k-3$

$b=8$

$c=2k+3$

$8^2-4(2k-3)(2k+3)<0$

$64-4(4k^2-9)<0$

$64-16k^2+36<0$

$-16k^2+100<0$

$100-16k^2<0$ as required

$M=100$

$N=16$

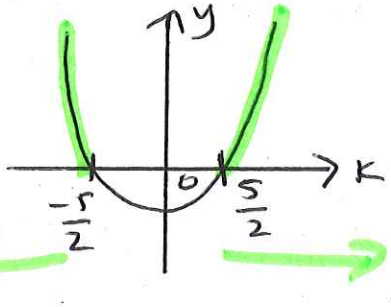
'x' -1 gives $16k^2-100>0$ ← careful with inequality sign

$\div 4$ $4k^2-25>0$

$(2k-5)(2k+5)>0$ ← difference of 2 squares

If we drew $y=(2k-5)(2k+5)$

crosses k-axis at $k=\frac{5}{2}$, $k=-\frac{5}{2}$



we require $(2k-5)(2k+5)>0$

$\therefore$ $k<-\frac{5}{2}$ or $k>\frac{5}{2}$

10. Given that the quadratic equation

$$(k-1)x^2 + 2kx + (7k-4) = 0$$

has no real roots, show that

$$6k^2 - 11k + 4 > 0.$$

Find the range of values of k satisfying this inequality. (7)

(Summer 14)

10. If no real roots discriminant < 0

$b^2 - 4ac < 0$

$a = k-1$

$b = 2k$

$c = 7k-4$

$(2k)^2 - 4(k-1)(7k-4) < 0$

$4k^2 - 4(7k^2 - 11k + 4) < 0$

$4k^2 - 28k^2 + 44k - 16 < 0$

$-24k^2 + 44k - 16 < 0 \quad (\div 4)$

$-6k^2 + 11k - 4 < 0 \quad (\div -1)$

(careful with inequality sign)

$6k^2 - 11k + 4 > 0$

'x' ac	'y' b
24	−11

$-8, -3$

$6k^2 - 11k + 4 > 0$

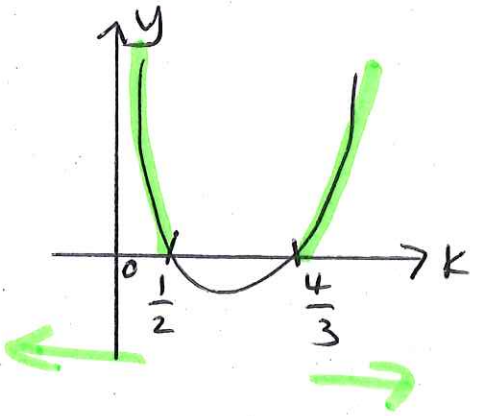
$6k^2 - 3k - 8k + 4 > 0$

$3k(2k-1) - 4(2k-1) > 0$

$(2k-1)(3k-4) > 0$

If we drew $y = (2k-1)(3k-4)$ where does it cross the k-axis?

$k = \frac{1}{2} \quad k = \frac{4}{3}$



We require, $(2k-1)(3k-4) > 0$

$k < \frac{1}{2}$ or $k > \frac{4}{3}$

11. a) Find the range of values of k for which the quadratic equation

$$kx^2 + (2k - 5)x + (k - 6) = 0$$

has **no real roots**. (4)

b) Without carrying out any further calculation, write down the value of k for which the quadratic equation

$$kx^2 + (2k - 5)x + (k - 6) = 0$$

has **two equal roots**. (1)

(Summer 15)

11 a) $kx^2 + (2k-5)x + (k-6) = 0$

no real roots

$\therefore$ discriminant < 0

$b^2 - 4ac < 0$

$a = k$

$b = 2k - 5$

$c = k - 6$

$(2k-5)^2 - 4(k)(k-6) < 0$

$4k^2 - 20k + 25 - 4k^2 + 24k < 0$

$4k + 25 < 0$

$4k < -25$

$k < -\frac{25}{4}$

b) equal roots, means discriminant is zero

$k = -\frac{25}{4}$

12. a) Find the range of values of k for which the quadratic equation

$$9x^2 + 8x - 2k = 0$$

has **two distinct real roots**. (4)

b) Solve the inequality

$$x(5x - 7) \geq 6.$$ (4)

(Summer 16)

12a) Two distinct roots

discriminant > 0

$b^2 - 4ac > 0$

$a = 9$

$b = 8$

$c = -2k$

$64 - 4(9)(-2k) > 0$

$64 + 72k > 0$

$72k > -64$

$k > \frac{-64}{72}$, $k > -\frac{8}{9}$

b) $x(5x-7) \geq 6$

$5x^2 - 7x - 6 \geq 0$

$+ac$	$+b$
-30	-7

$-10, 3$

$5x^2 - 10x + 3x - 6 \geq 0$

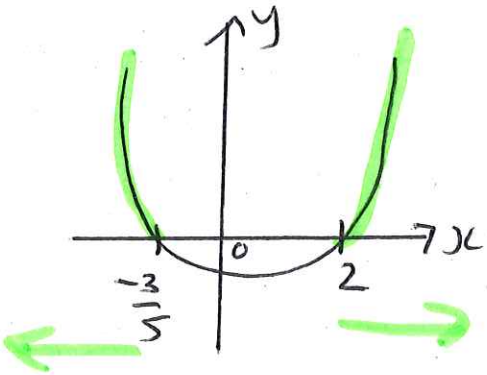
$5x(x-2) + 3(x-2) \geq 0$

$(x-2)(5x+3) \geq 0$

If we drew $y = (x-2)(5x+3)$

where would it cut the x-axis?

$x = 2 \quad x = -\frac{3}{5}$



we require

$(x-2)(5x+3) \geq 0$

$x \geq 2$ or $x \leq -\frac{3}{5}$

13. Solve the inequality $2x^2 + 11x + 12 \geq 0$. (3)

(Summer 17)

$2x^2 + 11x + 12 \geqslant 0$

$a = 2$
$b = 11$
$c = 12$

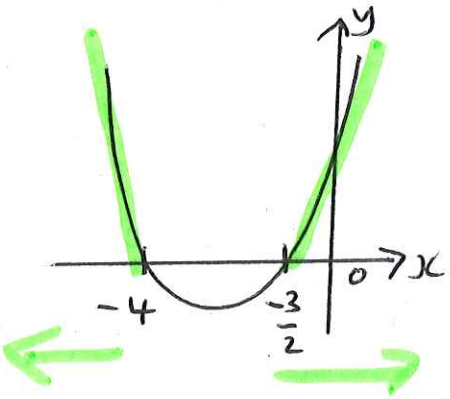
'x' ac	(+) b
24	11

8, 3

$2x^2 + 8x + 3x + 12 \geqslant 0$

$2x(x+4) + 3(x+4) \geqslant 0$

$(x+4)(2x+3) \geqslant 0$



If we drew $y = (x+4)(2x+3)$

cuts x axis at $x = -4$, $x = -\frac{3}{2}$

We require

$(x+4)(2x+3) \geqslant 0$

$x \leqslant -4$ or $x \geqslant -\frac{3}{2}$