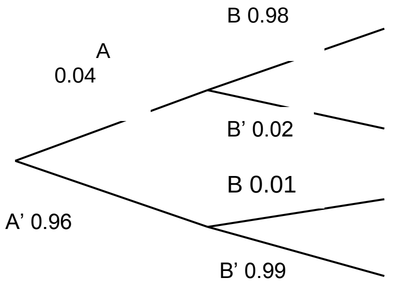
A2 Mathematics Unit 4: Applied Mathematics B

Solutions and Mark Scheme

SECTION A – Statistics

Qu. No.	Solution	Mark	AO	Notes
1(a)	 	M1	AO1	diagram
	A = the event that a person has the disease. B = the event that a positive response is obtained			
	Prob = $0.96 \times 0.99 = 0.9504$	A1	AO2	
	Alternative mark scheme for (a):			
	Prob = 0.96×0.99 $= 0.9504$	(M1) (A1)	(AO1) (AO2)	
(b)	$P(B) = 0.04 \times 0.98 + 0.96 \times 0.01$ $= 0.0488$	M1 A1	AO3 AO1	
(c)	$P(A\|B) = \frac{P(A \cap B)}{P(B)}$			
	$= \frac{0.04 \times 0.98}{0.0488}$	M1	AO3	
	$= 0.803(278688\ldots)$	A1	AO1	
		[6]		

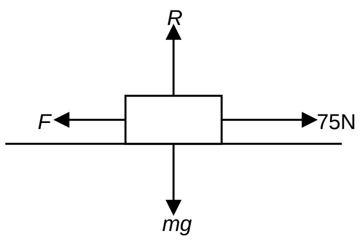
Qu. No.	Solution	Mark	AO	Notes
2(a)(i)	P(J wins with 1st shot) =P(M misses) × P(J hits)	M1	AO1	
	= 0.75*p*	A1	AO1	
(ii)	J wins with his second shot if the first three shots miss and then J hits the target with his second shot.	M1	AO3	
	P(J wins with 2nd shot) = 0.75 × (1 – *p*) × 0.75 × *p*	A1	AO2	
(b)	P(J wins game) = $0.75p + 0.75^2(1-p)p + 0.75^3(1-p)^2p + ...$	M1	AO3	
	Attempting to sum an infinite geometric series	M1	AO3	
	$= \dfrac{0.75p}{1-0.75(1-p)}$	A1	AO2	
	$= \dfrac{3p}{1+3p}$			
(c)	Mary is more likely to win if			
	$\dfrac{3p}{1+3p} < 0.5$	M1	AO3	
	leading to $p < \dfrac{1}{3}$	A1	AO1	
		[9]		
3(a)	Continuous uniform distribution on [30,60]	B1	AO3	
	Mean = 45	B1	AO1	
	Variance = 75	B1	AO1	
(b)	$P(\pi R^2 > 100) = P\left(R > \sqrt{\dfrac{100}{\pi}}\right)$	M1	AO3	
	$= P\left(L > 2\pi\sqrt{\dfrac{100}{\pi}}\right)$	A1	AO2	
	$= P(L > 35.45)$	A1	AO1	
	$= \dfrac{60-35.45}{30} = 0.818(\dot{3})$ or $\dfrac{491}{600}$	A1	AO1	
		[7]		

Qu. No.	Solution	Mark	AO	Notes
4(a)	Bell shaped	B1	AO2	Or Most values cluster in the middle of the range and the rest taper off symmetrically toward either extreme B0 for symmetrical only
(b)	1- P(6.12 < X < 8.12) = 1- 0.9949(0744) = 0.0051 (or 0.51%)	M1 A1	AO3 AO1	Or P(X < 6.12) + P(X > 8.12) M1A0 For 0.9949(0744)
(c)(i)	The population of weights of 2p coins is normally distributed. Mean and median in the sample are very similar, suggesting a symmetric distribution.	B1 B1	AO2 AO2	B1B0 The weights of 2p coins are normally distributed. Population must be stated or implied.
(ii)	H_o: The mean weight of all 2p coins in this batch = 7.12g H_1: The mean weight of all 2p coins in this batch < 7.12g (one-sided) *p*-value = P($\bar{x} < 6.89 \mid \mathrm{H}_0$) $= \mathrm{P}\left(z < \frac{6.89-7.12}{\frac{0.357}{\sqrt{30}}}\right)$ $= \mathrm{P}(z < -3.52(874))$ = 0.00021 (allow 0.00022) Since *p*-value<0.01, Reject H_o Very strong evidence to suggest the mean weight of the batch of 2p coins is less than 7.12(g)	B1 M1 A1 A1 A1 E1	AO3 AO1 AO1 AO1 AO2 AO3	Or H_o: μ= 7.12g B0 for H_o: Mean = 7.12g Population must be stated or implied, ie. the batch of 2p coins FT two-sided test *p*-value = 2 × 0.00021 = 0.00042
	Alternative Solution: TS = $\frac{6.89-7.12}{\frac{0.357}{\sqrt{30}}}$ = -3.52(874) CV = -2.32(63) Since TS< CV Reject H_o Very strong evidence to suggest the mean weight of the batch of 2p coins is less than 7.12(g)	(M1) (A1) (A1) (A1) (E1) **[11]**	(AO1) (AO1) (AO1) (AO2) (AO3)	FT Two-sided test CVs = ± 2.576 Since TS< - 2.576

Qu. No.	Solution	Mark	AO	Notes
5(a)	H_o: $\rho = 0$ H_1: $\rho \neq 0$ two-sided	B1	AO3	H_o: $\rho = 0$ H_1: $\rho > 0$ one-sided Population stated or implied
	TS = 0.895	B1	AO1	TS = 0.895
	CV = ±0.4821	B1	AO1	CV = ±0.412
	Since TS>0.4821, Reject H_o	B1	AO2	Since TS>0.412, Reject H_o
	Strong evidence to suggest the correlation coefficient is greater than zero	E1	AO3	Strong evidence to suggest the correlation coefficient is greater than zero
(b)	P-value for correlation between Value for money and Cost per night is > 0.05	E1	AO2	
	Cost per night does not seem to be correlated to Value for money.	E1	AO2	
		[7]		

SECTION B – Differential Equations and Mechanics

Question Number	Solution	Mark	AO	Notes
6. (a)	$\mathbf{a} = \mathbf{F}/m = \frac{1}{4}(4\mathbf{i} - 12\mathbf{j})$	M1	AO3	
	$\mathbf{a} = \mathbf{i} - 3\mathbf{j}$			
	Use $\mathbf{v} = \mathbf{u} + \mathbf{a}t$, $\mathbf{u} = -\mathbf{i} + 4\mathbf{j}$, $\mathbf{a} = \mathbf{i} - 3\mathbf{j}$	M1	AO2	
	$\mathbf{v} = (-\mathbf{i} + 4\mathbf{j}) + 5(\mathbf{i} - 3\mathbf{j})$			
	$\mathbf{v} = 4\mathbf{i} - 11\mathbf{j}$	A1	AO1	
(b)	$s = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2 + 7\mathbf{i} - 26\mathbf{j}$	M1	AO2	position vector relative to initial position vector.
		m1	AO2	adding initial positionvector.
	$\mathbf{s} = 2(-\mathbf{i} + 4\mathbf{j}) + \frac{1}{2} \times 4 \times (\mathbf{i} - 3\mathbf{j}) + (7\mathbf{i} - 26\mathbf{j})$			
	$\mathbf{s} = 7\mathbf{i} - 24\mathbf{j}$	A1	AO1	
	$\lvert\mathbf{s}\rvert = \sqrt{7^2 + 24^2}$	m1	AO1	
		A1	AO1	
	$\lvert\mathbf{s}\rvert = 25$	**[8]**		
7. (a)	Attempt to resolve in 2 directions	M1	AO3	dimensionally correct equation, no omitted or extra forces
	$T_1 \cos 23^\circ = T_2 \cos 40^\circ$	A1	AO2	correct equation
	$T_1 \sin 23^\circ + T_2 \sin 40^\circ = 160$	A1	AO2	correct equation
	Attempt to solve simultaneously	m1	AO1	any valid method
	$T_1 = 137.56(028\ldots)$ (N)	A1	AO1	
	$T_2 = 165.29(707\ldots)$ (N)	A1	AO1	
(b)	Object modelled as particle	B1	AO3	
	Cable modelled as light strings	B1	AO3	
		[8]		

Question Number	Solution	Mark	AO	Notes
8. (a)	$\frac{\mathrm{d}P}{\mathrm{d}t} = kP$	M1	AO3	
	$\int \frac{\mathrm{d}P}{P} = \int k\, dt$	m1	AO2	separation of variables
	$\ln P = kt + C$	A1	AO1	correct integration
	when $t = 0$, $P = 10$ $C = \ln 10$	m1	AO2	
	$\ln \frac{P}{10} = kt$			
	$e^{kt} = \frac{P}{10}$	m1	AO2	
	$P = 10\, e^{kt}$	A1	AO1	
(b)	When $t = 1$, $P = 20$ $k = \ln 2$	M1	AO2	
	$t = \frac{\ln 0\cdot 1P}{\ln 2}$ When $P = 1000000$			
	$t = \frac{\ln 100000}{\ln 2}$	m1	AO1	
	$t = 16.61$ hours	A1 **[9]**	AO1	
9.				
	$R = mg = 12 \times 9.8$ (= 117.6 N)	B1	AO1	
	Maximum friction = μR	M1	AO3	used
	Maximum friction = $0.8 \times 12 \times 9.8$ (= 94.08N)	A1	AO1	
	Therefore frictional force = 75 (N)	B1	AO3	
	because Max friction > tractive force	E1 **[5]**	AO3	

Question Number	Solution	Mark	AO	Notes
10. (a)	$x = (V\cos\theta)t$	B1	AO1	
	$y = (V\sin\theta)t - \frac{1}{2}gt^2$	B1	AO1	
(b)	$y = 0$ for time of flight $t = \frac{2V\sin\theta}{g}$	M1	AO2	
	Range $R = V\cos\theta.\frac{2V\sin\theta}{g}$	m1	AO2	
	$R = \frac{V^2\sin 2\theta}{g}$	A1	AO2	
(c) (i)	At maximum range, $\sin 2\theta = 1$ $\theta = 45°$ $\frac{V^2}{g} = 392$	M1	AO3	oe
	$V = 62.0$ (ms^{-1})	A1	AO1	cao
(ii)	$t = \frac{2\times 62\cdot 0\times\sin 45}{g}$ $t = 8.95$ (s)	A1	AO1	cao
(iii)	Max height when $t = 4.47$ s, $y_{max} = 62.5\times\sin 45°\times 4.47 - \frac{1}{2}\times 9.8\times 4.47^2$	m1	AO2	
	$y_{max} = 98.1$ (m)	A1	AO1	cao
		[10]		