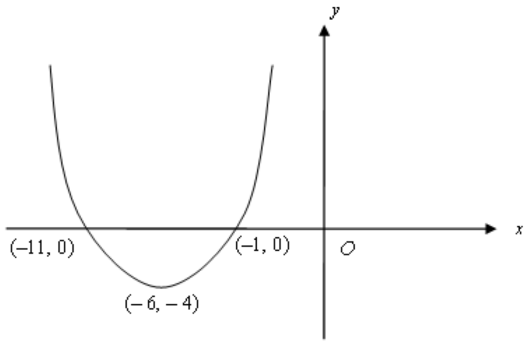
AS Mathematics Unit 1: Pure Mathematics A

Solutions and Mark Scheme

Question Number	Solution	Mark	AO	Notes
1. (a)	$A(1, -3)$	B1	AO1	
	A correct method for finding the radius, e.g., trying to rewrite the equation of the circle in the form $(x-a)^2 + (y-b)^2 = r^2$	M1	AO1	
	Radius = 5	A1	AO1	
(b)	Gradient $AP = \dfrac{\text{increase in } y}{\text{increase in } x}$	M1	AO1	
	Gradient $AP = \dfrac{(-7)-(-3)}{4-1} = -\dfrac{4}{3}$	A1	AO1	(f.t. candidate's coordinates for *A*)
	Use of $m_{\text{tan}} \times m_{\text{rad}} = -1$	M1	AO1	
	Equation of tangent is: $y-(-7) = \dfrac{3}{4}(x-4)$	A1 **[7]**	AO1	(f.t. candidate's gradient for *AP*)
2.	$7\sin^2\theta + 1 = 3(1-\sin^2\theta) - \sin^2\theta$	M1	AO1	(correct use of $\cos^2\theta = 1-\sin^2\theta$)
	An attempt to collect terms, form and solve a quadratic equation in $\sin\theta$, either by using the quadratic formula or by getting the expression into the form $(a\sin\theta + b)(c\sin\theta + d)$, with $a \times c$ = candidate's coefficient of $\sin^2\theta$ and $b \times d$ = candidate's constant	m1	AO1	
	$10\sin^2\theta + \sin\theta - 2 = 0$ $\Rightarrow (2\sin\theta + 1)(5\sin\theta - 2) = 0$ $\Rightarrow \sin\theta = -\dfrac{1}{2}, \sin\theta = \dfrac{2}{5}$	A1	AO1	(c.a.o.)
	$\theta = 210°, 330°$	B1 B1	AO1 AO1	
	$\theta = 23{\cdot}57(8178\ldots)°, 156{\cdot}42(182\ldots)°$	B1	AO1	
	Note: Subtract 1 mark for each additional root in range for each branch, ignore roots outside range. $\sin\theta$ = +, –, f.t. for 3 marks, $\sin\theta$ = –, –, f.t. for 2 marks $\sin\theta$ = +, +, f.t. for 1 mark	**[6]**		

Question Number	Solution	Mark	AO	Notes
3.	$y + k = (x + h)^3$	M1	AO2	
	$y + k = x^3 + 3x^2h + 3xh^2 + h^3$	A1	AO2	
	Subtracting y from above to find k	M1	AO2	
	$k = 3x^2h + 3xh^2 + h^3$	A1	AO2	
	Dividing by h and letting $h \to 0$	M1	AO2	
	$\frac{dy}{dx} = \lim_{h \to 0} \frac{k}{h} = 3x^2$	A1	AO2	(c.a.o.)
		[6]		
4.	Correct use of the Factor Theorem to find at least one factor *of* $f(x)$	M1	AO3	
	At least two factors of $f(x)$	A1	AO3	(accept $(x – 2·5)$ as a factor)
	$f(x) = (x + 3)(x – 4)(2x – 5)$	A1	AO3	(c.a.o.)
	Use of the fact that $f(x)$ intersects the y-axis when $x = 0$	M1	AO3	
	$f(x)$ intersects the y-axis at (0, 60)	A1	AO3	(f.t. candidate's expression for $f(x)$)
		[5]		
5. (a)	A correct method for finding the coordinates of the mid-point of *AB*	M1	AO1	
	D has coordinates *(– 1, 5)*	A1	AO1	
	Gradient of $AB = \frac{\text{increase in } y}{\text{increase in } x}$	M1	AO1	
	Gradient of $AB = -\frac{6}{2}$	A1	AO1	(or equivalent)
	Gradient of $CD = \frac{\text{increase in } y}{\text{increase in } x}$	(M1)	(AO1)	(to be awarded only if the previous M1 is not awarded)
	Gradient of $CD = \frac{7}{21}$	A1	AO1	(or equivalent)
	$-\frac{6}{2} \times \frac{7}{21} = -1 \Rightarrow$ *AB* is perpendicular to *CD*	B1	AO2	
(b)	A correct method for finding the length of *AD* or *CD*	M1	AO1	
	$AD = \sqrt{10}$	A1	AO1	
	$CD = \sqrt{490}$	A1	AO1	
	$\tan C\hat{A}B = \frac{CD}{AD}$	M1	AO1	
	$\tan C\hat{A}B = 7$	A1	AO1	
(c)	Isosceles	B1	AO2	
		[12]		

Question Number	Solution	Mark	AO	Notes
6. (a)	For statement A Choice of $c \neq -{}^{1}/_{2}$ and $d = -c - 1$ Correct verification that given equation is satisfied	 M1 A1	 AO2 AO2	
(b)	For statement B Use of the fact that any real number has an unique real cube root $(2c + 1)^3 = (2d + 1)^3 \Rightarrow 2c + 1 = 2d + 1$ $2c + 1 = 2d + 1 \Rightarrow c = d$	 M1 A1 A1 **[5]**	 AO2 AO2 AO2	
7. (a)				
	Concave up curve and y-coordinate of minimum = – 4 x-coordinate of minimum = – 6 Both points of intersection with x -axis	 B1 B1 B1	 AO1 AO1 AO1	
(b)	$y = -\frac{1}{2} f(x)$ **If B2 not awarded** $y = rf(x)$ with r negative	B2 (B1) **[5]**	AO2 AO2 (AO2)	

Question Number		Solution	Mark	AO	Notes
8.	(a)	A kite	B1	AO2	
	(b)	A correct method for finding $TR(TS)$	M1	AO3	
		$TR(TS) = \sqrt{96}$	A1	AO3	
		Area $OTR(OTS) = \frac{1}{2}\times\sqrt{96}\times 5$	M1	AO3	(f.t. candidate's derived value for $TR(TS)$)
		Area $OTRS = 2 \times$ Area $OTR(OTS)$	m1	AO3	
		Area $OTRS = 20\sqrt{6}$	A1	AO3	(c.a.o.)
			[6]		
9.		An expression for $b^2 - 4ac$ for the quadratic equation $4x^2 - 12x + m = 0$, with at least two of a, b or c correct	M1	AO1	
		$b^2 - 4ac = 12^2 - 4 \times 4 \times m$	A1	AO1	
		$b^2 - 4ac > 0$	m1	AO1	
		$(0<)\ m < 9$	A1	AO1	
		An expression for $b^2 - 4ac$ for the quadratic equation $3x^2 + mx + 7 = 0$, with at least two of a, b or c correct	(M1)		(to be awarded only if the corresponding M1 is not awarded above)
		$b^2 - 4ac = m^2 - 84$	A1	AO2	
		$m^2 < 81 \Rightarrow b^2 - 4ac < -3$	A1	AO2	
		$b^2 - 4ac < 0 \Rightarrow$ no real roots	A1	AO2	
			[7]		
10.	(a)	$(\sqrt{3} - \sqrt{2})^5 = (\sqrt{3})^5 + 5(\sqrt{3})^4(-\sqrt{2}) + 10(\sqrt{3})^3(-\sqrt{2})^2 + 10(\sqrt{3})^2(-\sqrt{2})^3 + 5(\sqrt{3})(-\sqrt{2})^4 + (-\sqrt{2})^5$	B2	AO1 AO1	(five or six terms correct)
		(**If B2 not awarded**, award B1 for three or four correct terms)			
		$(\sqrt{3} - \sqrt{2})^5 = 9\sqrt{3} - 45\sqrt{2} + 60\sqrt{3} - 60\sqrt{2} + 20\sqrt{3} - 4\sqrt{2}$	B2	AO1 AO1	(six terms correct)
		(**If B2 not awarded**, award B1 for three, four or five correct terms)			
		$(\sqrt{3} - \sqrt{2})^5 = 89\sqrt{3} - 109\sqrt{2}$	B1	AO1	(f.t. one error)
	(b)	Since $(\sqrt{3} - \sqrt{2})^5 \approx 0$, we may assume that $89\sqrt{3} \approx 109\sqrt{2}$	M1	AO3	(f.t candidate's answer to part (a) provided one coefficient is negative)
		Either: $89\sqrt{3} \times \sqrt{3} \approx 109\sqrt{2} \times \sqrt{3}$	m1	AO3	(f.t candidate's answer to part (*a*) provided one coefficient is negative)
		$\sqrt{6} \approx \frac{267}{109}$	A1	AO3	(c.a.o.)
		Or $89\sqrt{3} \times \sqrt{2} \approx 109\sqrt{2} \times \sqrt{2}$	(m1)	(AO3)	(f.t candidate's answer to part (*a*) provided one coefficient is negative)
		$\sqrt{6} \approx \frac{218}{89}$	(A1)	(AO3)	(c.a.o.)
			[8]		

Question Number	Solution	Mark	AO	Notes
11.	$a > 0$ $b > a + 2$ $b < 6 + 4a - a^2$	B1 B1 B1 **[3]**	AO1 AO1 AO1	
12.	Let $p = \log_a 19$, $q = \log_7 a$ Then $19 = a^p$, $a = 7^q$	B1	AO2	(the relationship between log and power)
	$19 = a^p = \left(7^q\right)^p = 7^{qp}$	B1	AO2	(the laws of indices)
	$qp = \log_7 19$			(the relationship between log and power)
	$\log_7 a \times \log_a 19 = \log_7 19$	B1 **[3]**	AO2	(convincing)
13. (a)	Choice of variable (x) for $AB \Rightarrow AC = x + 2$	B1	AO3	
	$(x+2)^2 = x^2 + 12^2 - 2 \times x \times 12 \times \frac{2}{3}$	M1	AO3	
	$x^2 + 4x + 4 = x^2 + 144 - 16x$	A1	AO3	
	$20x = 140 \Rightarrow x = 7$ $AB = 7$, $AC = 9$	A1	AO3	(Amend proof for candidates who choose $AC = x$)
(b)	$\sin A\hat{B}C = \frac{\sqrt{5}}{3}$	B1	AO1	
	$\frac{\sin B\hat{A}C}{12} = \frac{\sin A\hat{B}C}{9}$	M1	AO1	f.t. candidate's derived values for AC and $\sin A\hat{B}C$)
	$\sin B\hat{A}C = \frac{4\sqrt{5}}{9}$	A1 **[7]**	AO1	(c.a.o.)
14. (a)	Height of box $= \frac{9000}{2x^2}$	B1	AO3	(o.e.)
	$S = 2 \times (2x \times x + \frac{9000}{2x^2} \times x + \frac{9000}{2x^2} \times 2x$	M1	AO3	(f.t. candidate's derived expression for height of box in terms of x)
	$S = 4x^2 + \frac{27000}{x}$	A1	AO3	(convincing)
(b)	$\frac{dS}{dx} = 8x - \frac{27000}{x^2}$	B1	AO1	
	Putting derived $\frac{dS}{dx} = 0$	M1	AO1	
	$x = 15$	A1	AO1	(f.t. candidate's $\frac{dS}{dx}$)
	Stationary value of S at $x = 15$ is 2700	A1	AO1	(c.a.o)
	A correct method for finding nature of the stationary point yielding a minimum value	B1 **[8]**	AO1	

Question Number		Solution	Mark	AO	Notes
15.	(a)	A represents the initial population of the island.	B1	AO3	
	(b)	$100 = Ae^{2k}$ $160 = Ae^{12k}$	B1	AO1	(both values)
		Dividing to eliminate A	M1	AO1	
		$1{\cdot}6 = e^{10k}$	A1	AO1	
		$k = \frac{1}{10}\ln 1.6 = 0.047$	A1	AO1	(convincing)
	(c)	$A = 91({\cdot}0283)$	B1	AO1	(o.e.)
		When $t = 20$, $N = 91({\cdot}0283) \times e^{0{\cdot}94}$	M1	AO1	(f.t. candidate's derived value for A)
		$N = 233$	A1	AO3	(c.a.o.)
			[8]		
16.		$f'(x) = 3x^2 - 10x - 8$	M1	AO1	(At least one non-zero term correct)
		Critical values $x = -\frac{2}{3}, x = 4$	A1	AO1	(c.a.o)
		For an increasing function, $f'(x) > 0$	m1	AO1	
		For an increasing function $x < -\frac{2}{3}$ or $x > 4$	A2	AO2 AO2	(f.t. candidate's derived two critical values for x)
		Deduct 1 mark for each of the following errors the use of non-strict inequalities the use of the word 'and' instead of the word 'or'	**[5]**		

Question Number			Solution	Mark	AO	Notes
17.	(a)		$\frac{dy}{dx} = 3 - 2x$	M1	AO1	(At least one non-zero term correct)
			An attempt to find the value of $\frac{dy}{dx}$ at $x = 2$	m1	AO1	
			At $x = 2$, $\frac{dy}{dx} = -1$	A1	AO1	(c.a.o.)
			Equation of tangent at *B* is $y - 2 = -1(x - 2)$	A1	AO1	(f.t. candidate's value for $\frac{dy}{dx}$ at $x = 2$)
	(b)		x-coordinate of *A* = 3	B1	AO1	(derived)
			x-coordinate of *C* = 4	B1	AO1	(derived)
			If *D* is the foot of the perpendicular from *B* to the x-axis, area of triangle *BDC* = 2	B1	AO1	(f.t. candidate's derived x-coordinate of *C*)
			Area under curve = $\int_2^3 (3x - x^2)dx$	M1	AO3	(use of integration) (f.t. candidate's derived x-coordinate of *A*)
			$\frac{3x^2}{2} - \frac{x^3}{3}$	A1	AO3	(correct integration)
			Area under curve = (27/2 – 9) – (6 – 8/3)	m1	AO3	(an attempt to substitute limits, f.t. candidate's derived x-coordinate of *A*)
			Shaded area = Area of triangle *BDC* – Area under curve	m1	AO3	(f.t. candidate's derived x-coordinates of *A* and *C*)
			Shaded area = 5/6	A1 **[12]**	AO3	(c.a.o.)
18.	(a)	(i)	4**u** – 3**v** = 20**i** – 27**j**	B1 B1	AO1 AO1	
		(ii)	A correct method for finding the length of *UV*	M1	AO1	
			Length of *UV* = 10	A1	AO1	
	(b)	(i)	Position vector of $C = \frac{1}{10}\mathbf{a} + \frac{9}{10}\mathbf{b}$ or $C = \frac{9}{10}\mathbf{a} + \frac{1}{10}\mathbf{b}$	M1	AO3	
			Position vector $C = \frac{1}{10}\mathbf{a} + \frac{9}{10}\mathbf{b}$	A1	AO3	
		(ii)	The position vector of any point on the road will be of the form λ**a** + (1 – λ)**b** for some value of λ	B1 **[7]**	AO2	