TRANSFORMATIONS OF FUNCTIONS

AS Unit 1: Pure Mathematics A

WJEC past paper questions: 2010 – 2017

Total marks available 72 (approximately 1 hour 20 minutes)

1. Figure 1 shows a sketch of the graph $y = f(x)$. The graph has a maximum point at (2, 5) and intersects the x-axis at the points (-2, 0) and (6, 0).
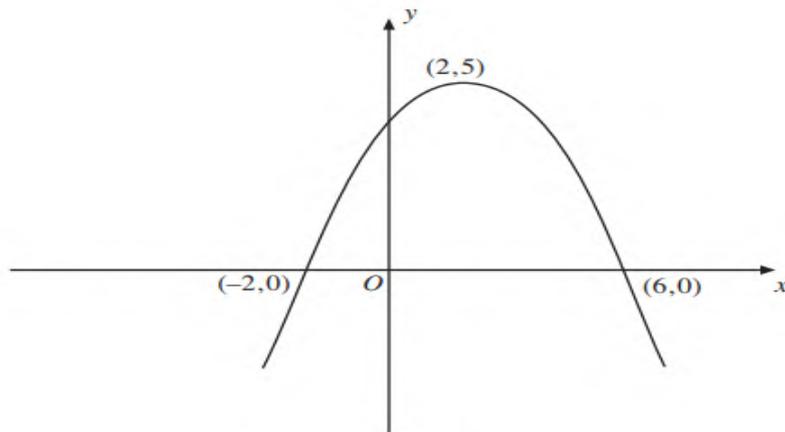

Figure 1

a) Sketch the graph of $y = f\left(\frac{x}{2}\right)$, indicating the coordinates of the stationary point and the coordinates of the points of intersection of the graph with the x-axis. (3)

b) Figure 2 shows a sketch of the graph having **one** of the following equations with an appropriate value of either p, q or r.

$$y = f(x + p), \text{ where } p \text{ is a constant}$$
$$y = f(x) + q, \text{ where } q \text{ is a constant}$$
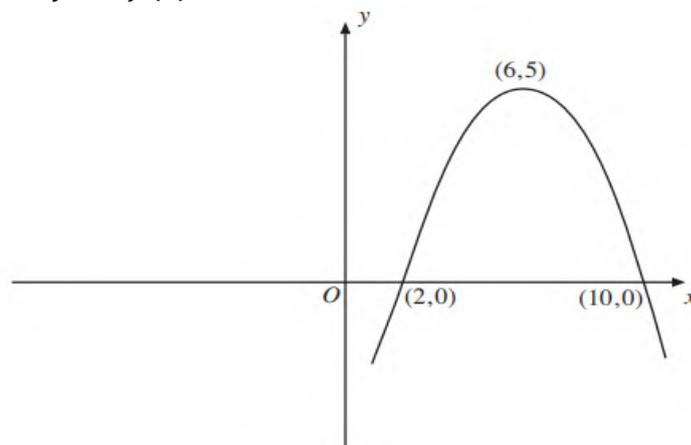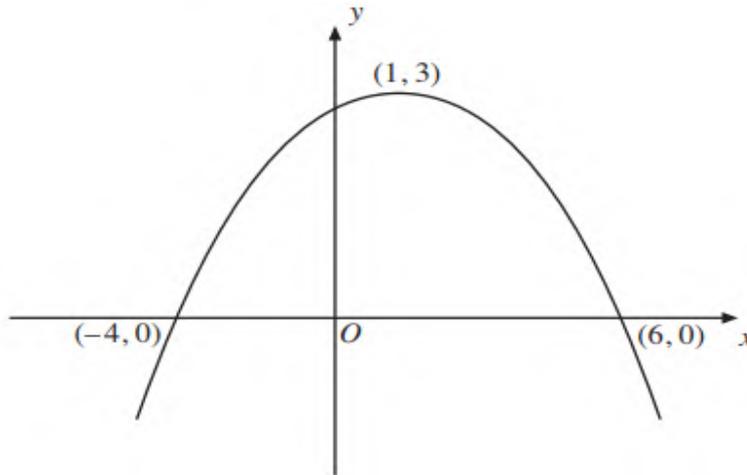$$y = rf(x), \text{ where } r \text{ is a constant}$$

Figure 2

Write down the equation of the graph sketched in Figure 2, together with the value of the corresponding constant. (2)

(January 10)

2. The diagram shows a sketch of the graph $y = f(x)$. The graph passes through the points (-4, 0) and (6, 0) and has a maximum point at (1, 3).
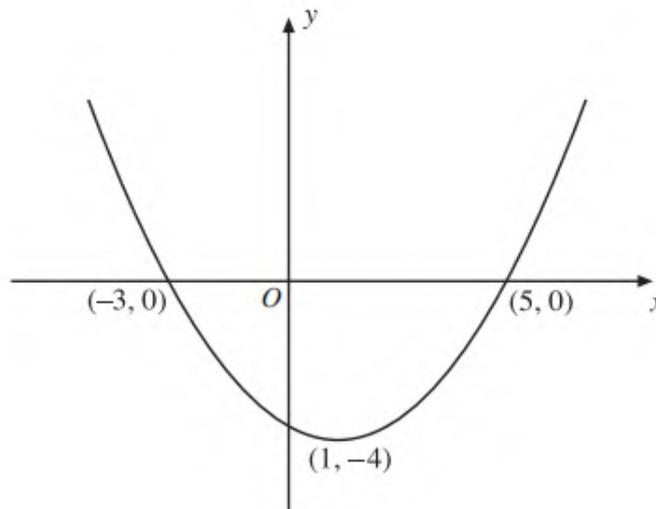

Sketch the following graphs, using a separate set of axes for each graph. In each case, you should indicate the coordinates of the stationary point and the coordinates of the points of intersection of the graph with the x-axis.

a) $y = 2f(x)$ (3)

b) $y = f(-x)$ (3)

(Summer 10)

3. The diagram shows a sketch of the graph $y = f(x)$. The graph passes through the points (-3, 0) and (5, 0) and has a minimum point at (1, -4).

Sketch the following graphs, using a separate set of axes for each graph. In each case, you should indicate the coordinates of the stationary point and the coordinates of the points of intersection of the graph with the x-axis.

a) $y = f(x + 3)$ (3)

b) $y = -f(x)$ (3)

(January 11)

4. Figure 1 shows a sketch of the graph $y = f(x)$. The graph has a minimum point at (-3, -4) and intersects the x-axis at the points (-8, 0) and (2, 0).
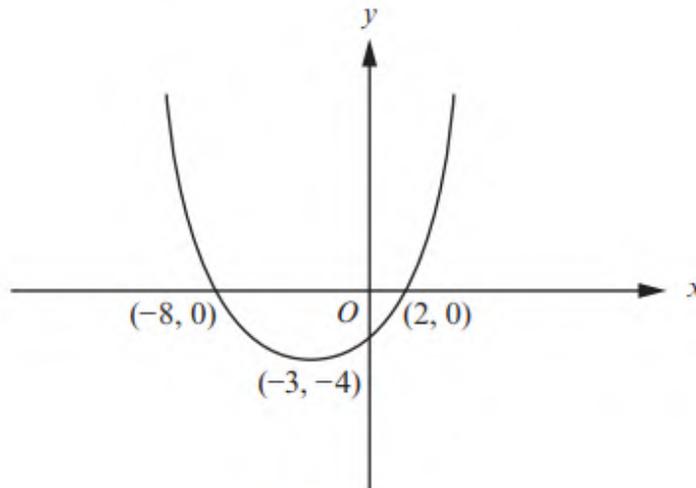

Figure 1

a) Sketch the graph of $y = f(x + 3)$, indicating the coordinates of the stationary point and the coordinates of the points of intersection of the graph with the x-axis. (3)

b) Figure 2 shows a sketch of the graph having **one** of the following equations with an appropriate value of either p, q or r.

$$y = f(px), \text{where } p \text{ is a constant}$$
$$y = f(x) + q, \text{where } q \text{ is a constant}$$
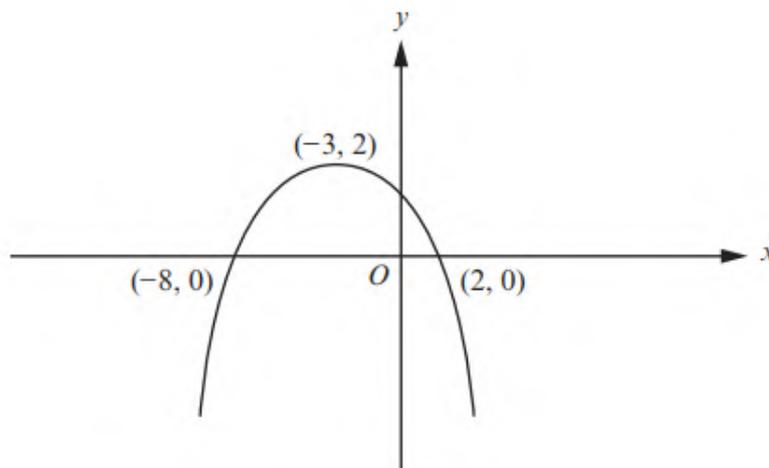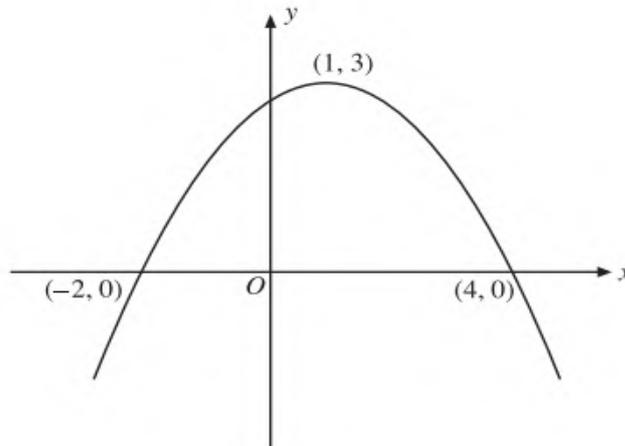$$y = rf(x), \text{where } r \text{ is a constant}$$

Figure 2

Write down the equation of the graph sketched in Figure 2, together with the value of the corresponding constant. (2)

(Summer 11)

5. The diagram shows a sketch of the graph $y = f(x)$. The graph has a maximum point at (1, 3) and intersects the x-axis at the points (-2, 0) and (4, 0).
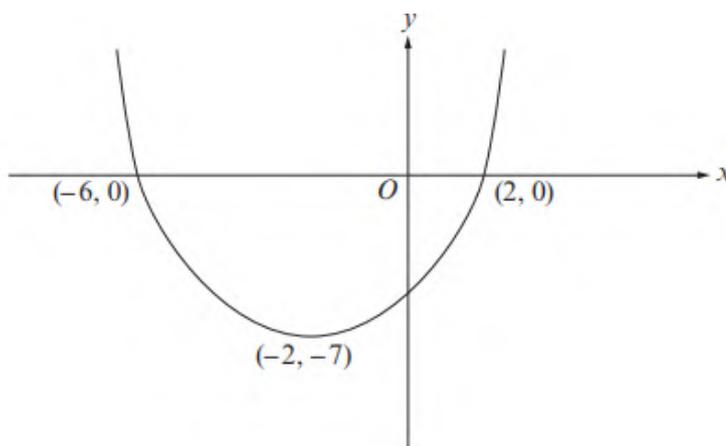

a) Sketch the graph of $y = f(2x)$, indicating the coordinates of the stationary point and the coordinates of the points of intersection of the graph with the x-axis. (3)

b) i) Sketch the graph of $y = f(x) - 5$, indicating the coordinates of the stationary point.

ii) Given that f is a quadratic function, use the graph you have drawn in part i) to write down the number of real roots of the equation
$$f(x) - 5 = 0.$$ (3)

(January 12)

6. The diagram shows a sketch of the graph $y = f(x)$. The graph passes through the points (-6, 0) and (2, 0) and has a minimum point at (-2, -7).
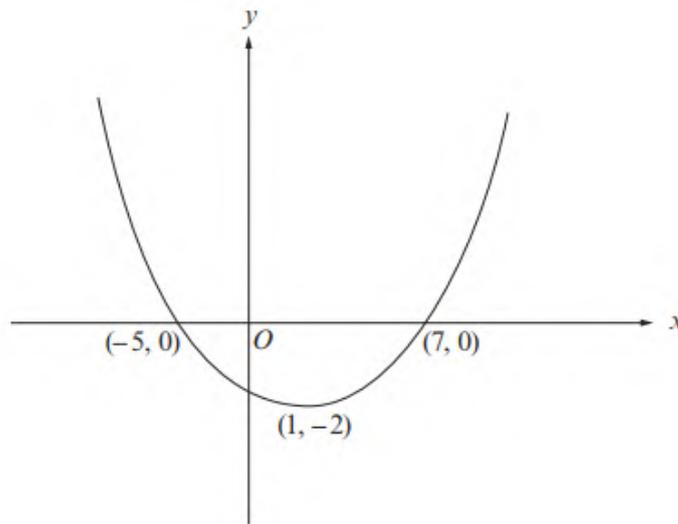

Sketch the following graphs, using a separate set of axes for each graph. In each case, you should indicate the coordinates of the stationary point and the coordinates of the points of intersection of the graph with the x-axis.

a) $y = f(x - 5)$ (3)

b) $y = f\left(\dfrac{x}{2}\right)$ (3)

(Summer 12)

7. The diagram shows a sketch of the graph $y = f(x)$. The graph passes through the points (-5, 0) and (7, 0) and has a minimum point at (1, -2).
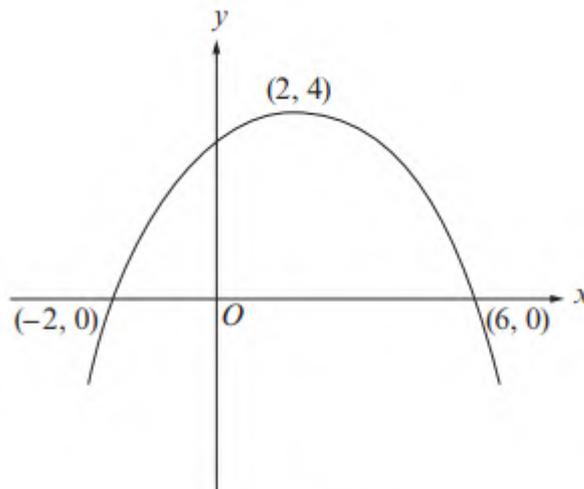
    .

Sketch the following graphs, using a separate set of axes for each graph. In each case, you should indicate the coordinates of the stationary point and the coordinates of the points of intersection of the graph with the x-axis.

a) $y = 3f(x)$ (3)

b) $y = f(-x)$ (3)

          (January 13)

8. The diagram shows a sketch of the graph $y = f(x)$. The graph passes through the points (-2, 0) and (6, 0) and has a maximum point at (2, 4).

Sketch the following graphs, using a separate set of axes for each graph. In each case, you should indicate the coordinates of the stationary point and the coordinates of the points of intersection of the graph with the x-axis.

a) $y = f(x + 5)$ (3)

b) $y = f(-2x)$ (3)

          (Summer 13)

9. **Figure 1** shows a sketch of the graph $y = f(x)$. The graph has a maximum point at (2, 6) and intersects the x-axis at the points (-4, 0) and (8, 0).
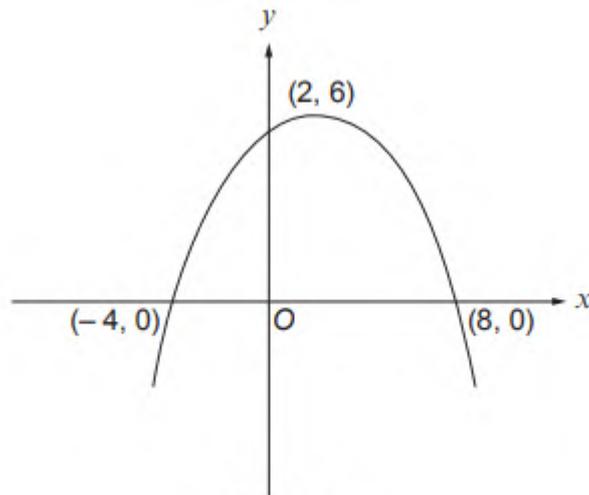

Figure 1

a) Sketch the graph of $y = f(x - 3)$, indicating the coordinates of the stationary point and the coordinates of the points of intersection of the graph with the x-axis.　　(3)

b) **Figure 2** shows a sketch of the graph having **one** of the following equations with an appropriate value of either p, q or r.

$y = f(x) + p$, where p is a constant

$y = f(qx)$, where q is a constant
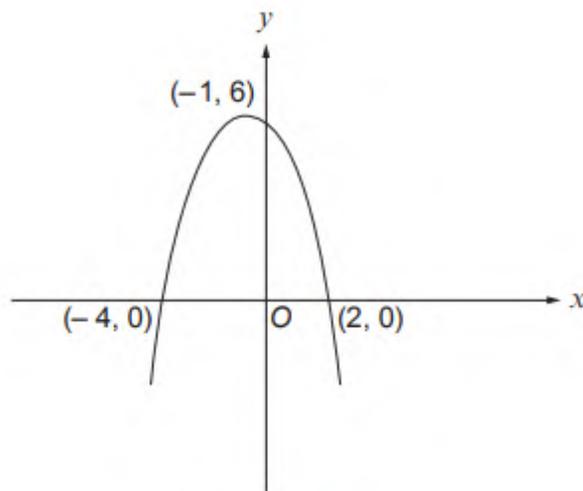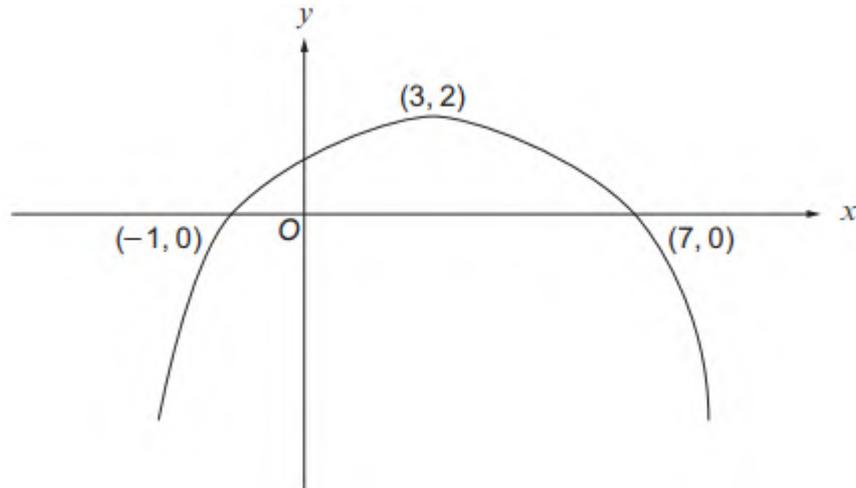
$y = rf(x)$, where r is a constant

Figure 2

Write down the equation of the graph sketched in **Figure 2**, together with the value of the corresponding constant.　　(2)

(January 14)

10. The diagram shows a sketch of the graph $y = f(x)$. The graph passes through the points (-1, 0) and (7, 0) and has a maximum point at (3, 2).

a) Sketch the following graphs, using a separate set of axes for each graph. In each case, you should indicate the coordinates of the stationary point and the coordinates of the points of intersection of the graph with the x-axis.
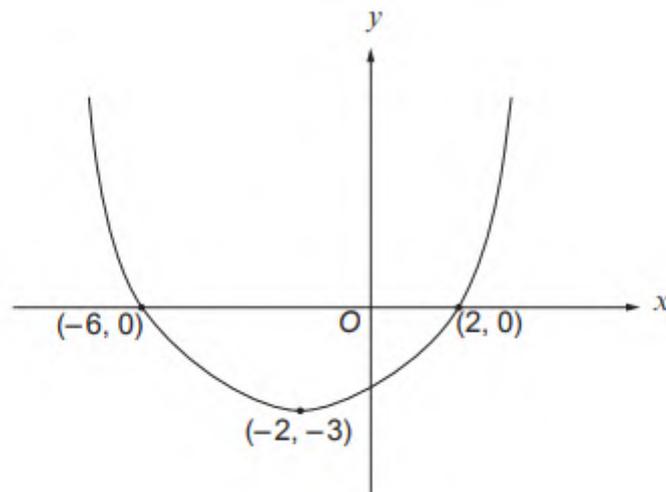   i) $y = f(x + 4)$
   ii) $y = -2f(x)$ (6)
b) Hence write down one root of the equation
   $$f(x + 4) = -2f(x) + 4.$$ (1)

(Summer 14)

11. The diagram shows a sketch of the graph $y = f(x)$. The graph passes through the points (-6, 0) and (2, 0) and has a minimum point at (-2, -3).

a) Sketch the graph of $y = f\left(\frac{1}{2}x\right)$, indicating the coordinates of the stationary point and the coordinates of the points of intersection of the graph with the x-axis. (3)
b) Angharad is asked by her teacher to draw the graph of $y = af(x)$ for various non-zero values of the constant a. One of Angharad's graphs passes through the origin O. Explain why this cannot possibly be correct. (1)

(Summer 15)

12. Figure 1 shows a sketch of the graph $y = f(x)$. The graph has a minimum point at (1, -3) and intersects the x-axis at the points (-4, 0) and (6, 0).
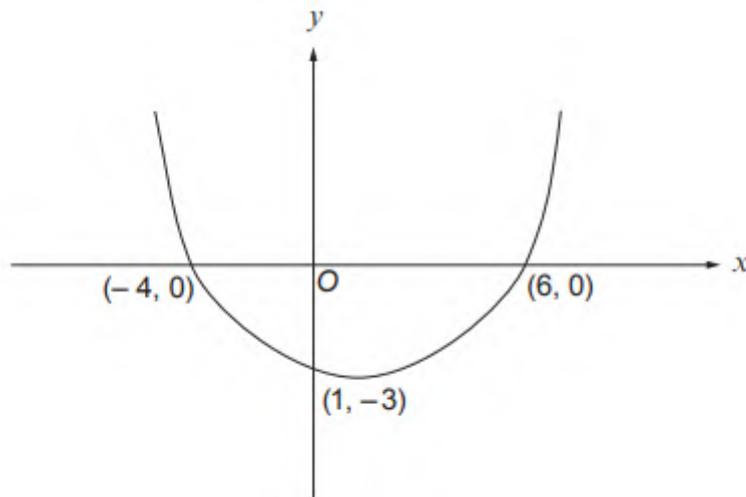

Figure 1

a) Sketch the graph of $y = -3f(x)$, indicating the coordinates of the stationary point and the coordinates of the points of intersection of the graph with the x-axis. (3)

b) **Figure 2** shows a sketch of the graph $y = g(x)$, where

$$g(x) = f(x) + p, \text{ where } p \text{ is a constant,}$$
$$\text{or} \quad g(x) = f(qx), \text{ where } q \text{ is a constant,}$$
$$\text{or} \quad g(x) = rf(x), \text{ where } r \text{ is a constant,}$$
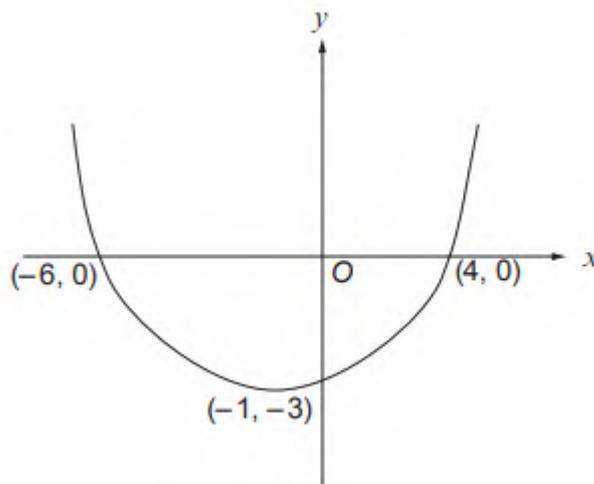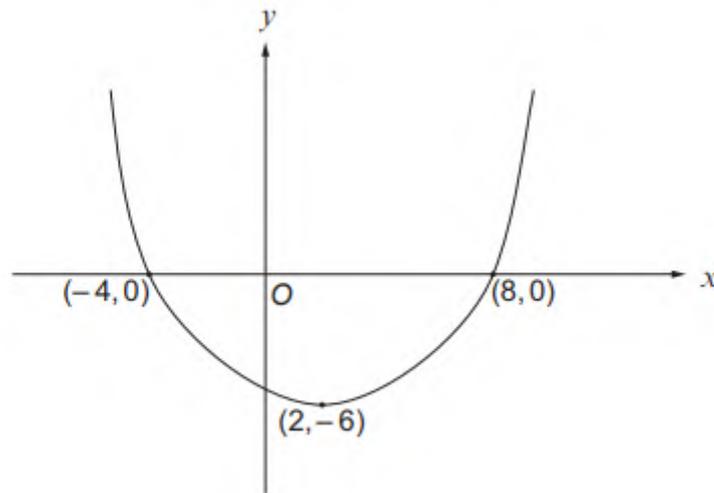$$\text{or} \quad g(x) = f(x + s), \text{ where } s \text{ is a constant.}$$

Figure 2

The function g can in fact be any one of **two** of the above functions. In each of these two cases, write down the expression for $g(x)$, including the value of the corresponding constant. (2)

(Summer 16)

13. The diagram shows a sketch of the graph $y = f(x)$. The graph passes through the points (-4, 0) and (8, 0) and has a minimum point at (2, -6).
hyguukh

a) Sketch the graph of $y = -\frac{1}{2}f(x)$, indicating the coordinates of the stationary point and the coordinates of the points of intersection of the graph with the x-axis. (3)

c) Siân is asked by her teacher to draw the graph of $y = f(ax)$ for various non-zero values of the constant a. Write down two facts about the stationary point on Siân's graph which will always be true whatever her choice of a. (2)

(Summer 17)