POISSON DISTRIBUTION

AS Unit 2: Applied Mathematics A

Section A: Statistics

WJEC past paper questions: 2010 - 2017

Total marks available 98 (approximately 2 hours)

1. Sheila buys two biased dice in a shop. Each time either dice is thrown, the probability of obtaining a six is 0.2.
   a) She throws one of the dice 50 times. Determine the probability that she obtains
   i) exactly 12 sixes,
   ii) at least 10 sixes.   (5)
   b) She now throws the two dice simultaneously 200 times. Use a Poisson approximation to find the probability that between 5 and 10 (both inclusive) double sixes are obtained .   (5)
   (Summer 10)

2. The number of customers arriving at a village shop during an interval of length t minutes may be assumed to have a Poisson distribution with mean $0.1t$.
   a) Find the probability that the number of customers arriving between 10 a.m. and 11 a.m. is
   i) exactly 3,
   ii) less than 5.   (5)
   b) Given that the probability of no customers arriving during an interval of t minutes is equal to 0.25, find the value of t correct to two decimal places.   (4)
   (Summer 10)

3. Wine glasses are packed in boxes, each containing 20 glasses. Each glass has a probability of 0.05 of being broken in transit, independently of all other glasses.
   a) Let X denote the number of glasses in a box broken in transit.
   i) State the distribution of X.
   ii) **Without** the use of tables, calculate $P(X = 1)$.
   iii) **Using tables**, determine the value of $P(X \geq 3)$.   (5)
   b) A retailer buys 10 of these boxes. Use a Poisson approximation to find the probability that less than 5 of the 200 glasses are broken in transit.   (3)
   (January 11)

4. Cars arrive at a petrol station in such a way that the number arriving during an interval of length *t* minutes has a Poisson distribution with mean 0.2*t*.
    a) Find the probability that
    i)   exactly ten cars arrive between 9 a.m. and 10 a.m. ,
    ii) more than five cars arrive between 11 a.m. and 11. 30 a.m..   (6)
    b) The probability that no cars arrive during an interval of length *t* minutes is equal to 0.03. Without the use of tables, find the value of *t*.   (4)

    (Summer 11)

5. The number of emergency admissions, X, into a hospital during each 24-hour period can be modelled by a Poisson distribution with mean 3.6.
    a) **Without the use of tables**, determine
    i) $P(X = 5)$,
    ii) $P(X < 3)$.   (5)
    b) **Using tables**, determine $P(3 \leq X \leq 7)$.   (3)

    (January 12)

6. Jim sells jars of honey at a Saturday market. The demand each Saturday for his jars can be modelled by a Poison distribution with mean 12.
    a) Find the probability that the demand on a randomly chosen Saturday is
    i)   exactly 10 jars,
    ii) more than 10 jars.   (4)
    b) Jim wants the probability of being able to satisfy the demand for his honey to be at least 0.95. Find the minimum number of jars that he needs to take to the market.   (2)

    (Summer 12)

7. a) When a certain type of seed is planted, there is a probability of 0.7 that it produces red flowers. A gardener plants 20 of these seeds. Calculate the probability that
    i)   exactly 15 seeds produce red flowers,
    ii) more than 12 seeds produce red flowers.   (6)
    b) When a different type of seed is planted, there is a probability of 0.09 that it produces white flowers. The gardener plants 150 of these seeds. Use an appropriate Poisson distribution to determine, approximately, the probability that exactly 10 seeds produce white flowers.   (3)

    (January 13)

8. A newsagent sells the Daily Bugle newspaper. You may assume that the daily demand for this newspaper has a Poisson distribution with mean 15. The newspaper begins each day with 20 copies of the newspaper.
    a) Calculate the probability that, on a randomly chosen day, the newsagent sells
    i)   12 copies of the newspaper,
    ii) all 20 copies of the newspaper.   (4)
    b) Determine the minimum number of copies of the Daily Bugle that the newsagent should buy each day in order to satisfy the demand with a probability of at least 0.99.
                                      (2)

    (January 14)

9. In a junior football match, it may be assumed that the number of goals scored in any interval of length t minutes has a Poisson distribution with mean $0.1t$. **Without the use of tables**, find the probability that the number of goals scored in the first 15 minutes of play is

   a) 2, (3)

   b) more than 2. (3)

   (Summer 14)

10. Customers arrive at a shop such that the number of arrivals in a time interval of duration t minutes follows a Poisson distribution with mean $0.2t$.

   a) Without the use of tables, determine the probability that the number of arrivals between 10:00 a.m. and 10:30a.m. is

   i) exactly 5,

   ii) more than 3. (6)

   b) Given that the probability of exactly 5 arrivals in an interval of duration t minutes is 0.0602, where $t < 30$, use tables to determine the value of t. (3)

   (Summer 16)

11. Jane is solving a problem in which she has to calculate $P(X = 2)$ where X has a Poisson distribution with mean 3. Unfortunately, she has no statistical tables with her and her simple calculator has no e^x button and it can only carry out arithmetic operations. She decides to use an appropriate binomial distribution to give an approximate value for $P(X = 2)$. She takes $n = 50$.

   a) what value of p should she take? (2)

   b) Write down and evaluate an arithmetic expression giving her approximate value correct to four decimal places. (2)

   c) Show that the approximation is within 1% of the value obtained from the appropriate Poisson table. (3)

   (Summer 16)

12. Independently for each page, the number of typing errors per page in the first draft of a book has a Poisson distribution with mean 0.8.

   a) i) Calculate the probability that a randomly chosen page contains at least one error.
   ii) Calculate the probability that the third page of three randomly chosen pages is the first to contain at least one error. (4)

   b) i) Write down the probability p_n that a random selection of n pages contains no errors.
   ii) Find the smallest value of n such that $p_n < 0.001$. (6)

   (Summer 17)