BINOMIAL DISTRIBUTION

AS Unit 2: Applied Mathematics A

Section A: Statistics

WJEC past paper questions: 2010 - 2017

Total marks available 87 (approximately 1 hour 45 minutes)

1. When seeds of a certain variety of flower are planted, the probability of each seed germinating is 0.8, independently of all other seeds.
   a) David plants 20 of these seeds. Find the probability that
   i) exactly 15 seeds germinate,
   ii) at least 15 seeds germinate. (6)
   b) Beti plants *n* of these seeds and she correctly calculates that the probability that they all germinate is 0.10737, correct to five decimal places. Find the value of *n*. (3)
   (January 10)

2. a) A series of trials is carried out, each resulting in either success or failure. State **two** conditions that have to be satisfied in order for the total number of successes to be modelled by the binomial distribution. (2)
   b) Each time Ann shoots an arrow at a target, she hits it with probability 0.4. She shoots 20 arrows at the target. Determine the probability that she hits it
   i) exactly 8 times,
   ii) between 6 and 10 times (both inclusive). (5)
   c) Each time she shoots an arrow, she hits the centre of the target with probability 0.04. She shoots 100 arrows at the target. Use a Poisson approximation to find the probability that she hits the centre of the target less than 5 times. (3)
   (Summer 11)

3. Alun and Ben are snooker players. When they play a game against each other, Alun wins with probability 0.6 and successive games are independent.
   a) One evening they play 10 games against each other. Determine the probability that Alun wins
   i) exactly 7 games,
   ii) at least 6 games. (5)
   b) On another evening, find the probability that Alun wins for the first time on the fourth game. (3)
   (January 12)

4. Charlie and Dave regularly play chess against each other. When they play each other, Charlie wins with probability 0.75 and successive games are independent.
   a) One weekend they play 10 games against each other. Determine the probability that Charlie wins
   i) exactly 4 games,
   ii) more than 5 games. (5)
   b) The probability that a game lasts for less than one hour is 0.08. They play 45 games against each other over a holiday period. Use a Poisson approximation to determine the probability that more than 6 of these games lasts for less than one hour. (3)
   (Summer 12)

5. Bethan has two fair dice, each in the shape of a regular tetrahedron. The four faces of each dice are numbered 1, 2, 3, 4 respectively.
   a) She throws one of the dice 20 times and her score on each throw is defined as the number appearing on the face in contact with the table. Let X denote the number of throws resulting in a score of 4.
   i) Write down the distribution of *X.*
   ii) Determine $P(3 \leq X \leq 9)$.
   iii) **Without the use of tables**, calculate $P(X = 6)$. (6)
   b) She now throws the two dice simultaneously 160 times and her score on each throw is defined as the sum of the numbers on the two faces in contact with the table. Use a Poisson approximation to determine the probability that the number of throws resulting in a score of 8 is
   i) equal to 12,
   ii) between 6 and 14 (both inclusive). (6)
   (Summer 13)

6. a) The random variable *X* has the binomial distribution B (20, 0.2).
   i) Without the use of tables, calculate $P(X = 6)$.
   ii) Determine $P(2 \leq X \leq 8)$. (5)
   b) The random variable *Y* has the binomial distribution B (200, 0.0123). Use the Poisson distribution to determine the approximate value of $P(Y = 3)$. (3)
   (January 14)

7. A zoologist is studying a certain breed of dog.
   a) He knows from past experience that the probability of a newly born puppy being female is 0.55. He selects from a random sample of 20 newly born puppies. Calculate the probability that the number of females in the sample is
   i) exactly 12,
   ii) between 8 and 16 (both inclusive). (8)
   b) The probability of a newly born puppy being yellow is 0.05. Use an approximating distribution to find the probability that less than 5 out of a random sample of 60 newly born puppies are yellow. (3)
   (Summer 14)

8. a) A factory manufactures cups. The manager knows from past experience that 5% of the cups produced are defective. Given a random sample of 50 of these cups, determine the probability that the number of defective cups in this sample is

i) exactly 2,

ii) between 3 and 8 (both inclusive). (6)

b) The factory also manufactures plates. The manger knows that 1.5% of the plates produced are defective. Use an appropriate Poisson distribution to find, approximately, the probability that a random sample of 250 of these plates contain exactly 4 defective plates. (3)

(Summer 15)

9. In a shooting range at a country fair, customers pay £5 to fire 8 shots at a target. Let X denote the number of shots which hit the target. Prizes are awarded according to the following rules.

If $X < 2$, no prize awarded.

If $X = 2$, a prize of £10 is awarded.

If $X > 2$, a prize of £25 is awarded.

Jim decides to spend £5 to fire 8 shots. You may assume that the probability of one of his shots hitting the target is 0.12 and that successive shots are independent.

a) Calculate the probability that he wins

i) no prize,

ii) a £10 prize,

iii) a £25 prize. (5)

(Summer 16)

10. Anne and Brian play a board game against each other regularly.

a) The probability that Anne wins a game is 0.7 and the probability that Brian wins a game is 0.3, independently of all other games. One day, they play 10 games. Let X denote the number of games won by Anne on that day.

i) State the distribution of X, including any parameters.

ii) Find the probability that Anne wins more games than Brian. (4)

b) The probability that one of their games takes more than 1 hour to complete is 0.06. During a school holiday, they play 44 games. Use a Poisson approximation to find the probability that more than 2 of these games takes more than 1 hour to complete. (3)

(Summer 17)