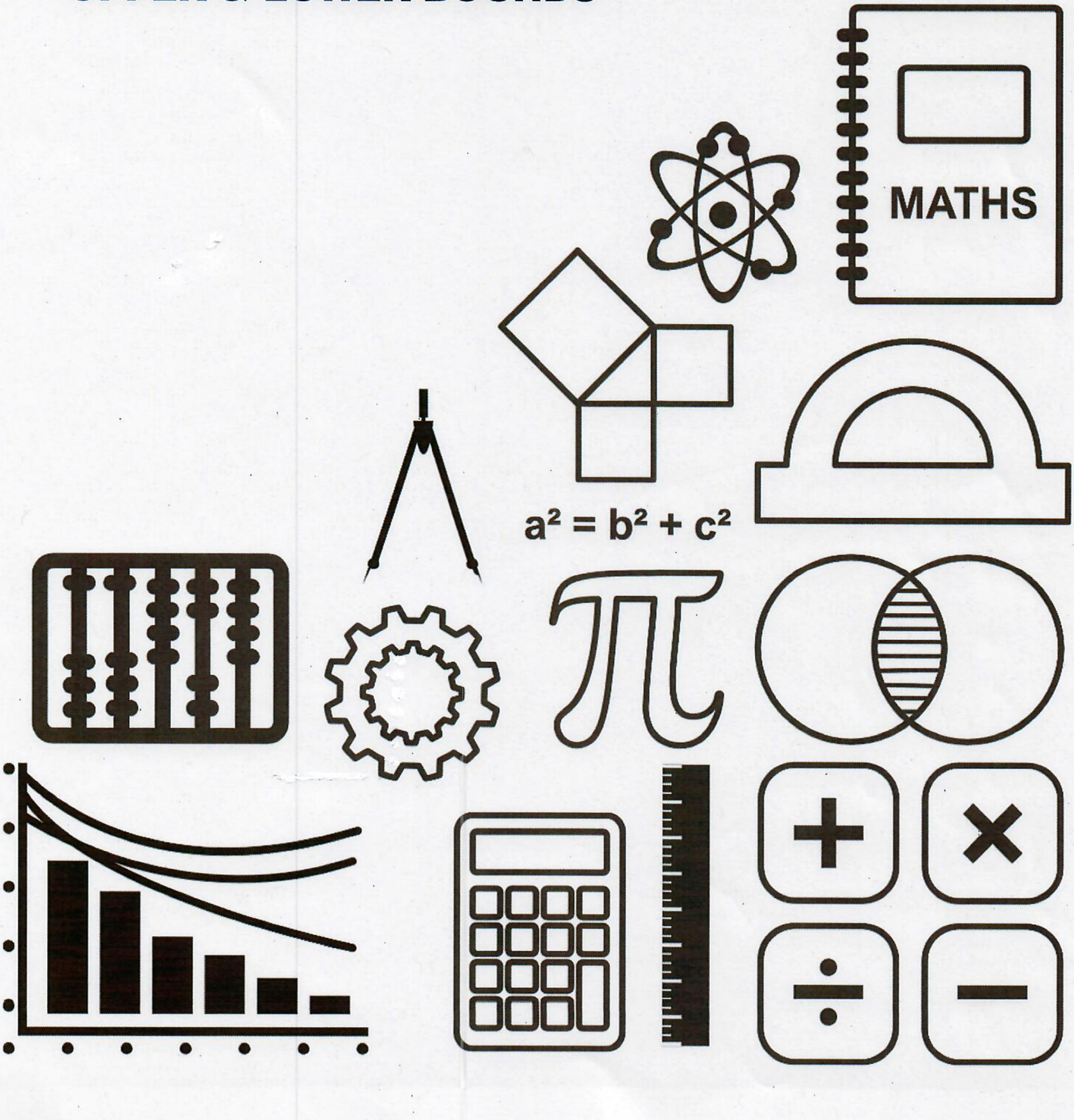
MATHSDIY

SOLUTIONS

GCSE TOPIC BOOKLET
UPPER & LOWER BOUNDS



MATHSDIY

1. The length of a plank of wood is 950 mm, measured to the nearest 10 mm.
Write down the **least** and **greatest** possible values of the length of the plank.

Least value 945 mm Greatest value 955 mm [2]

2. The side of a square tile is 20cm, measured to the nearest cm.
Write down the least and the greatest lengths of the side of the tile.

Least length 19.5 cm Greatest length 20.5 cm

[2]

3. The length of a roll of plastic sheeting is 500 cm, measured to the nearest 5 cm.

(a) Write down the least possible length and greatest possible length of the roll of plastic sheeting.

Least possible length 497.5 cm Greatest possible length 502.5 cm [2]

(b) A plastic sheet of length 100 cm, measured to the nearest 5 cm, is cut from the roll of plastic sheeting. Find the least possible length of the sheeting left on the roll.

Least possible length left if

1) sheeting was originally as small as possible

= 497.5

& 2) piece cut off was as big as possible = 102.5cm

Sheeting left = 497.5 − 102.5 = 395cm

[3]

4. A truck can carry a load of 1600 kg, to the nearest 100 kg. A crate weighs 75 kg, correct to the nearest 5 kg. What is the maximum possible number of crates that could be placed on the truck?

If lorry carries 1650kg and crates weigh 72.5kg

This is the 'best case' scenario to carry as many as possible.

1650 ÷ 72.5 = 22.758... 22 crates

[4]

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5. Two boxes are stacked one on top of the other.
The height of one box is 57 cm correct to the nearest centimetre.
The height of the other box is 38 cm correct to the nearest centimetre.
Find the least height and the greatest height of the boxes when stacked one on top of the other.

Least height = 56·5 + 37·5 = 94 cm
Greatest height = 57·5 + 38·5 = 96 cm

Least height 94 cm

Greatest height 96 cm

[2]

6.

A DIY store sells lengths of kitchen worktops. There are two different suppliers of the worktops. One supplier, "Worktop Magic", states that the length of each worktop is 4000 mm, measured to the nearest 5 mm. The other supplier, "Worktops 4 U", states that the length of each worktop is 4000 mm ± 3 mm.

(a) Complete the following table.

	Least possible length	Greatest possible length
Worktop Magic	3997.5 mm	4002·5 mm
Worktops 4 U	3997 mm	4003 mm

[3]

(b) A customer requires a worktop to fit a length of at least 4·02 m. Would Worktop Magic or Worktops 4 U be able to supply a suitable worktop? Give a reason for your answer.

4·02m ×1000 = 4020mm
Maximum from worktop magic is 4002·5mm.
Maximum from worktops 4U is 4003mm.
Both supplies 'biggest possible' worktops are too small. No.

[3]

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7. A company makes paving slabs.
The dimensions of the paving slabs are:

Length 0·8 metres — 80cm — 79.5cm – 80.5cm
Width 0·55 metres — 55cm — 54.5cm – 55.5cm

Each dimension is measured correct to the nearest centimetre.

The company makes this claim in an advertisement:

100 paving slabs will cover an area of at least 43·5 m²

(a) Is this claim true? Show your working and give a reason for your answer.

Minimum area of one paving slab
$= 0.795 \times 0.545 = 0.433275\ m^2$
If we had 100 of these, $100 \times 0.433275\ m^2$
$= 43.3275\ m^2$ ∴ If all 100 slabs were the least dimensions, NO the claim is FALSE.
$43.3275 < 43.5$

[4]

(b) Write down the maximum area that could be covered by 100 of these paving slabs.

$(0.805 \times 0.555) \times 100 = 44.6775\ m^2$

[2]

8. Two boxes have heights of 134 mm and 23 mm, each measured to the nearest mm.
Find the maximum height when the boxes are placed one on top of the other.

$134.5 + 23.5 = 158\ mm$

[1]

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9. Square paving stones are manufactured so that the length of each of them is 51 cm, measured to the nearest centimetre.

(a) Write down the least and greatest length of a square paving stone.

Least length 50.5 cm Greatest length 51.5 cm

[2]

(b) The length of a path is 30·0 m, measured correct to the nearest 0·1 of a metre. Starting at one end of the path, 60 of the paving stones are laid end to end in a straight line along the path. Explain, showing all your calculations, why the paving stones will always be longer than the path.

We must prove that even if the paving stones are SMALL AS POSSIBLE & the path is LONG AS POSSIBLE the paving stones will still be longer.

$60 \times 50.5 = 3030\text{cm} \div 100 = 30.3\text{m}$

maximum path = 30.05m

∴ The paving stones are always longer than the path. $30.3\text{m} > 30.05\text{m}$

[5]

10. A jug has a volume of 1000 cm^3, measured to the nearest 50 cm^3.

(a) Write down the least possible value of the volume of the jug and the greatest possible value of the volume of the jug.

Least possible volume 975 cm^3 Greatest possible volume 1025 cm^3

[2]

Water is poured from the jug into a tank of volume 52 litres measured to the nearest litre.

(b) Explain, showing all your calculations, why it is always possible to pour water from 50 full jugs into the tank without overflowing.

We must prove that even if the jug is as big as possible & the tank is as small as possible it will not overflow.

50 x greatest jugs = $50 \times 1025 = 51250\text{cm}^3 \div 1000 = 51.25$ litres

Smallest tank = 51.5 litres

∴ The tank will never overflow. $51.25 < 51.5$

[5]

MATHSDIY

11. Fertiliser is sold in bags, each of mass 12 kg, correct to the nearest kg.

(a) Write down the least possible value and the greatest possible value of the mass of a bag.

Least possible mass ....11.5.... kg Greatest possible mass ....12.5.... kg

[2]

(b) (i) Find the least possible value and the greatest possible value of the mass of 100 bags.

$100 \times 11.5 = 1150$ kg

$100 \times 12.5 = 1250$ kg

Least mass of 100 bags ....1150.... kg Greatest mass of 100 bags ....1250.... kg

[2]

(ii) Chris wishes to be sure that he delivers 1200 kg of fertiliser to a customer.
Find the least number of bags Chris needs to deliver in order to be sure that at least 1200 kg of fertiliser is delivered.

Assuming 'worst case scenario' that all the bags delivered are as small as possible

$1200 \div 11.5 = 104.348\ldots$ bags

105 bags

[3]

Make sure you understand why you must round up to 105 here and not down.

MATHSDIY

12.

Wall tiles for sale

Length 30 cm **Width 15 cm**

All measurements to the nearest centimetre

(a) Is it always possible to tile an area of up to $8500\,\text{cm}^2$ using 20 of these tiles?
You must give a reason and show your working.

Worst case scenario are that all of the tiles are as small as possible.

Least width = 14.5cm Least length = 29.5cm

Least area = $14.5 \times 29.5 = 427.75\,\text{cm}^2$

20 of these = $20 \times 427.75 = 8555\,\text{cm}^2$

Yes. Even if all of the tiles are as small as possible you can tile $8500\,\text{cm}^2$.

[4]

(b) Write down the maximum area that could be covered using 20 of these tiles.

$20 \times (30.5 \times 15.5) = 9455\,\text{cm}^2$

[2]

MATHSDIY

13. A time of 35·2 seconds, measured correct to the nearest tenth of a second, was recorded for the winner of a 300 metres race.
The race track had been marked out to within an accuracy of ±0·1%.
Clearly explaining your reasoning, calculate the greatest and least possible values of the average speed of the winner, giving your answers in metres per second.

$35.15s - 35.25s$

$\frac{0.1}{100} \times 300m = 0.3m$

$299.7m - 300.3m$

$$\text{Average Speed} = \frac{\text{Total distance}}{\text{Total time}}$$

$$\underline{\underline{\text{Least Average speed}}} = \frac{\text{least total distance}}{\text{greatest total time}}$$

$$= \frac{299.7}{35.25} = \underline{\underline{8.502..}}$$

$$= \underline{\underline{8.50m/s}}$$

$$\underline{\underline{\text{Greatest Average speed}}} = \frac{\text{greatest total distance}}{\text{least total time}}$$

$$= \frac{300.3}{35.15} = \underline{8.543\cdots}$$

$$= \underline{\underline{8.54m/s}}$$

[7]

14.

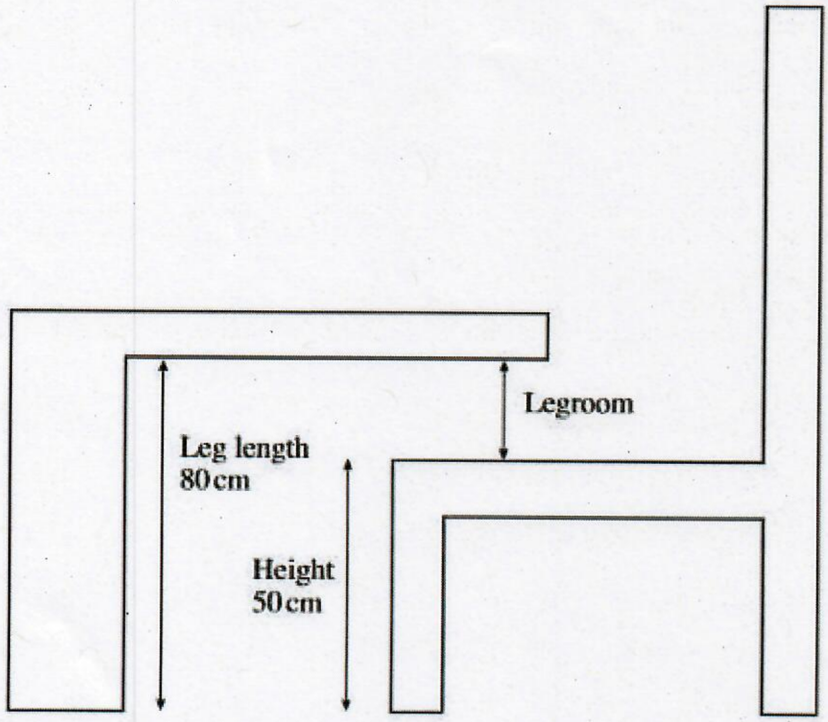


Diagram not drawn to scale.

The legroom between a table and a chair is calculated by finding the difference between the length of the leg of a table and the height of the chair. In the diagram, both the height of the chair and the length of the leg of the table are given correct to the nearest cm.
Find, in centimetres, the least and greatest possible values of the legroom.

Greatest leg room when chair height minimum & table height maximum $= 80.5 - 49.5 = 31$ cm

least leg room when chair height maximum & table height minimum $= 79.5 - 50.5 = 29$ cm

Least 29 cm Greatest 31 cm

[4]

15. The length of a table top is 2050 mm, measured to the nearest 10 mm.

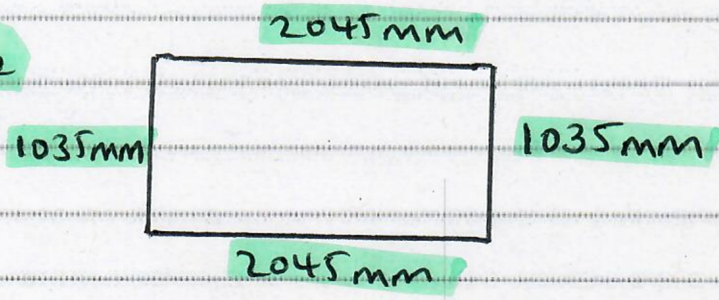
(a) Write down the least and greatest possible values of the length of the table top.

Least value 2045 mm   Greatest value 2055 mm

[2]

(b) The width of the table top is 1040 mm, measured to the nearest 10 mm. Find the least possible perimeter of the table top.

least possible perimeter



$= 2045 + 1035 + 2045 + 1035$

$= 6160\text{mm}$

[3]