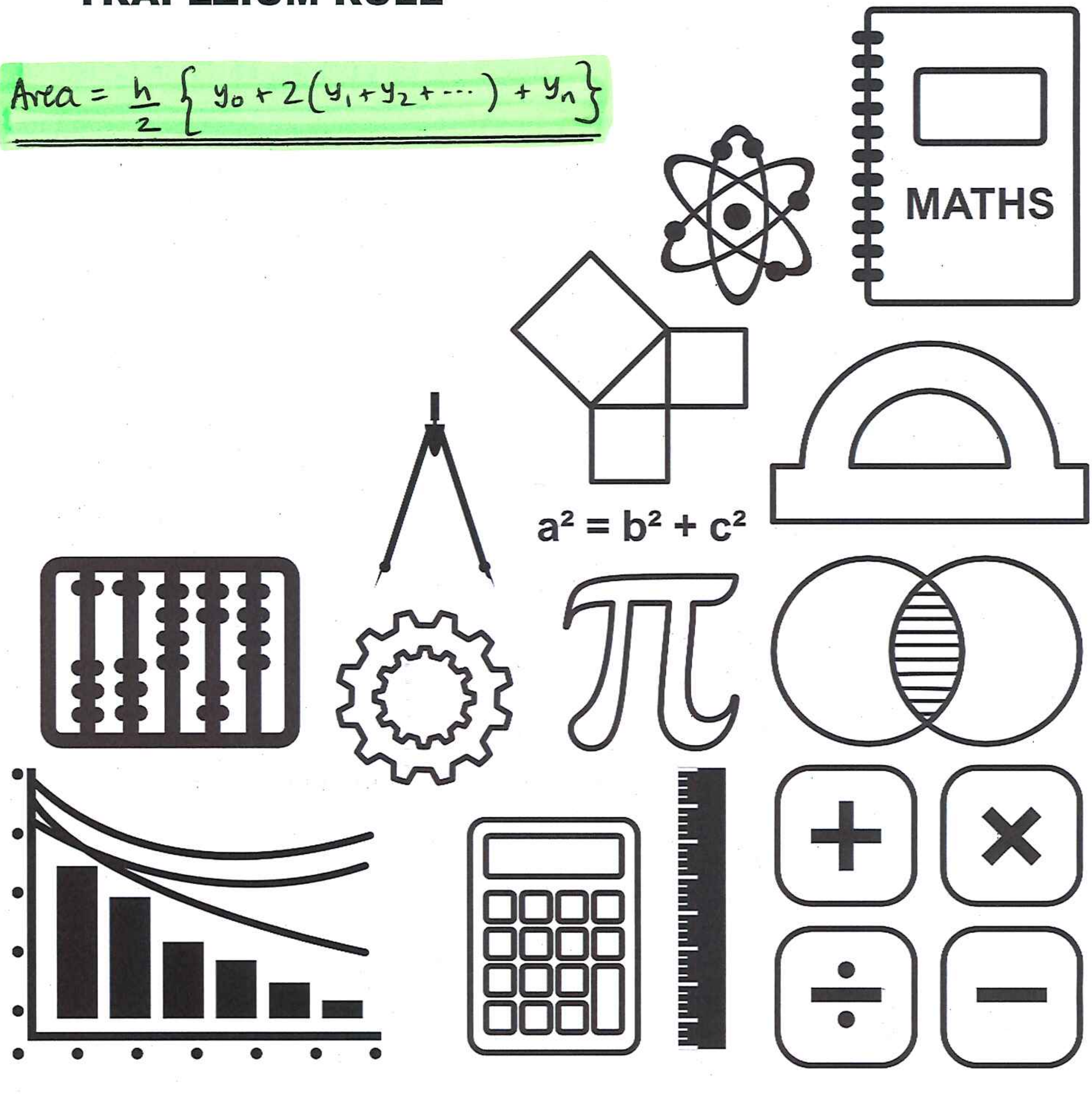
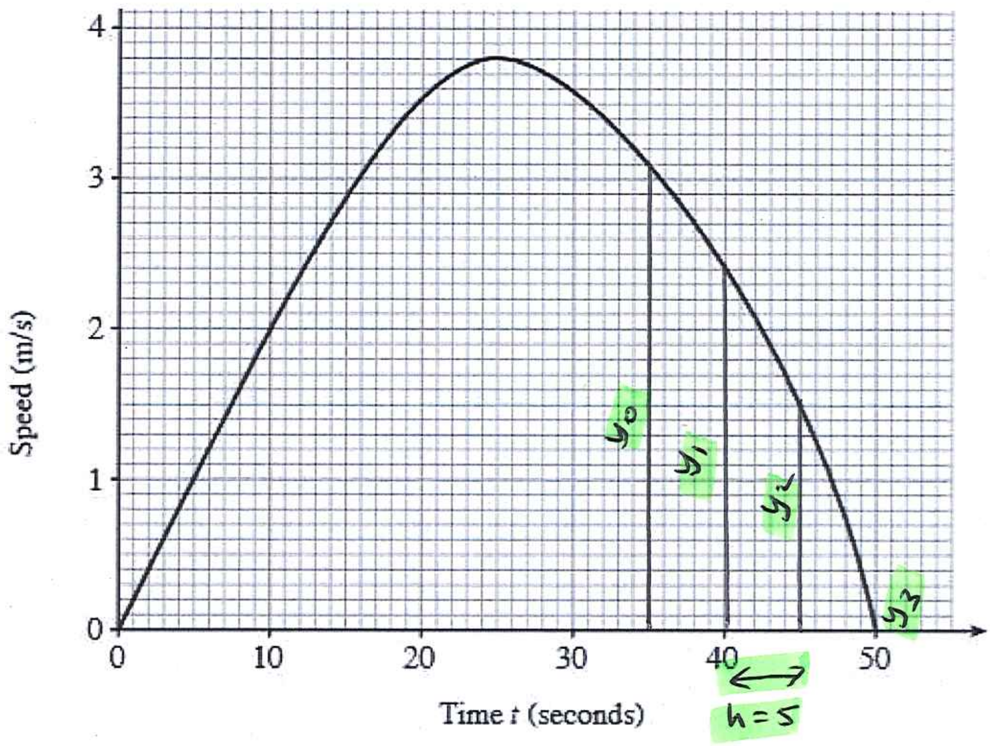
MATHSDIY

GCSE TOPIC BOOKLET
TRAPEZIUM RULE

$$\text{Area} = \frac{h}{2}\left\{ y_0 + 2(y_1 + y_2 + \cdots) + y_n \right\}$$



1. The graph below shows the speed of a train, in m/s, over a period of 50 seconds starting at time $t = 0$ seconds.



The table below gives the speed of the train between $t = 35$ and $t = 50$.

Time t (seconds)	35	40	45	50
Speed (m/s)	3·1	2·4	1·5	0
	y_0	y_1	y_2	y_3

distance = area under a speed/time graph

Use the trapezium rule with the values taken from the table to estimate the distance, in metres, travelled by the train between $t = 35$ and $t = 50$.

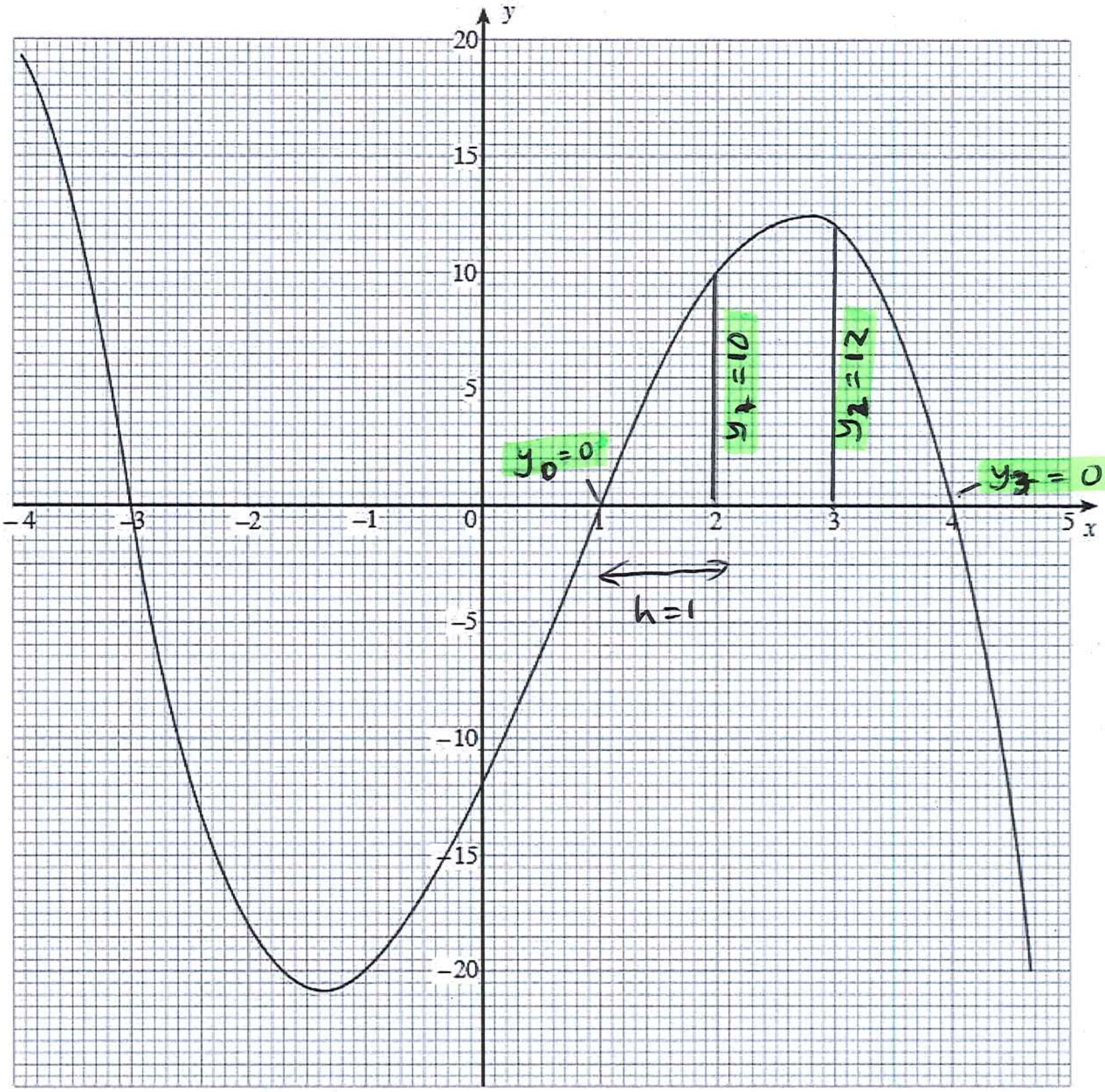
$$\text{Area} = \frac{h}{2}\left\{y_0 + 2(y_1 + y_2) + y_3\right\}$$

$$= \frac{5}{2}\left\{3.1 + 2(2.4 + 1.5) + 0\right\}$$

$$= \underline{27.25 \text{ metres}}$$

[3]

2. The graph of $y = -x^3 + 2x^2 + 11x - 12$ is shown below.



Use the trapezium rule, with ordinates $x = 1$, $x = 2$, $x = 3$ and $x = 4$, to find an estimate for the area of the region enclosed by the curve $y = -x^3 + 2x^2 + 11x - 12$ and the x-axis between the values $x = 1$ and $x = 4$.

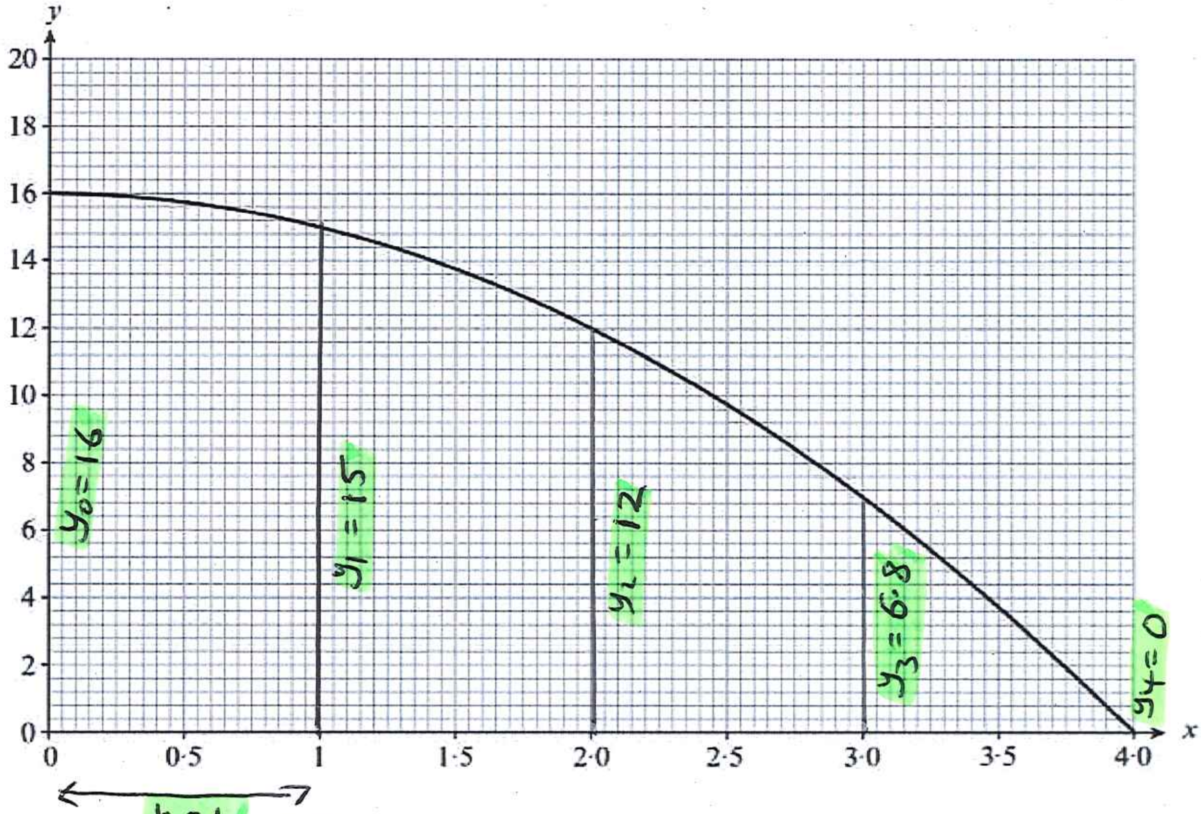
$$\text{Area} = \frac{h}{2}\left\{y_0 + 2(y_1 + y_2) + y_3\right\}$$

$$= \frac{1}{2}\left\{0 + 2(10 + 12) + 0\right\}$$

$$= \underline{\underline{22 \text{ units}^2}}$$

[4]

3. A sketch of $y = 16 - x^2$ is shown below for values of x from 0 to 4.



Use the trapezium rule, with the five ordinates $x = 0$, $x = 1$, $x = 2$, $x = 3$ and $x = 4$, to estimate the area of the region bounded by the curve, the x-axis and the y-axis.

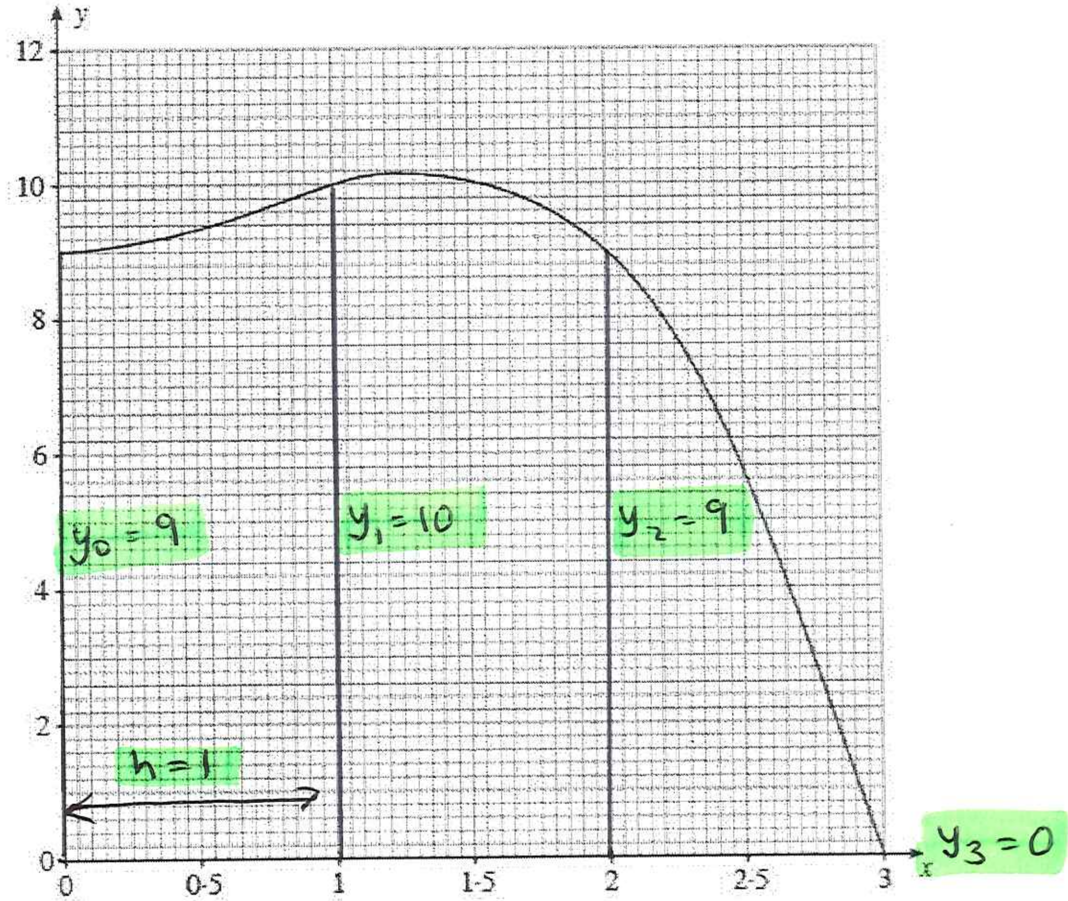
$$\text{Area} = \frac{h}{2}\left\{y_0 + 2(y_1 + y_2 + y_3) + y_4\right\}$$

$$= \frac{1}{2}\left\{16 + 2(15 + 12 + 6 \cdot 8) + 0\right\}$$

$$= \underline{\underline{41 \cdot 8 \text{ units}^2}}$$

[4]

4. A sketch of $y = 9 + 2x^2 - x^3$ is shown in the diagram below for values of x from $x = 0$ to $x = 3$.



The following table of values of $y = 9 + 2x^2 - x^3$ for $x = 0, 1, 2$ and 3.

x	0	1	2	3
y	9	10	9	0

Use the values from the table and the trapezium rule with three strips to calculate an estimate for the area of the region bounded by the curve, the x-axis and the y-axis.

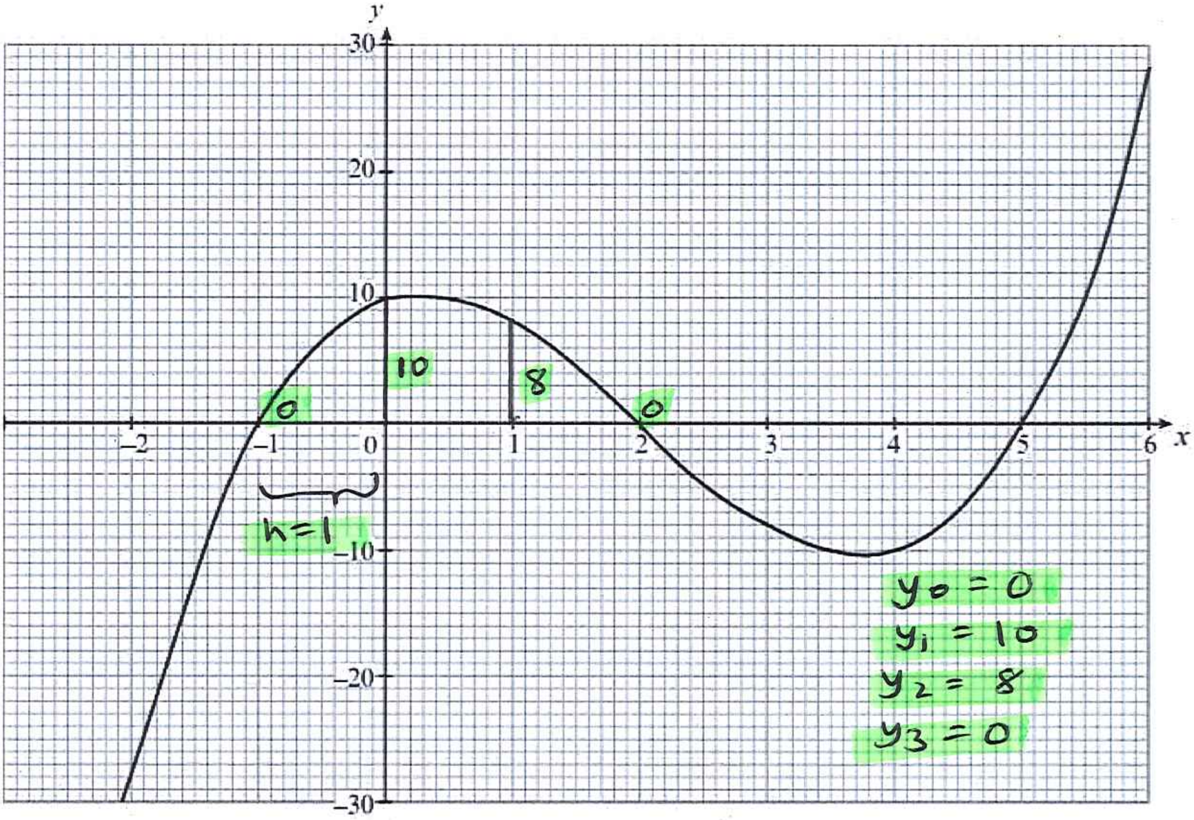
$$\text{Area} = \frac{h}{2}\left\{y_0 + 2(y_1 + y_2) + y_3\right\}$$

$$= \frac{1}{2}\left\{9 + 2(10 + 9) + 0\right\}$$

$$= \frac{1}{2}(47) = 23{\cdot}5 \text{ units}^2$$

[3]

5. The graph of $y = x^3 - 6x^2 + 3x + 10$, for values of x between $x = -2$ and $x = 6$, is drawn below.



Use the trapezium rule with three strips of equal width to estimate the area of the region enclosed by the curve $y = x^3 - 6x^2 + 3x + 10$ and the x-axis between $x = -1$ and $x = 2$.

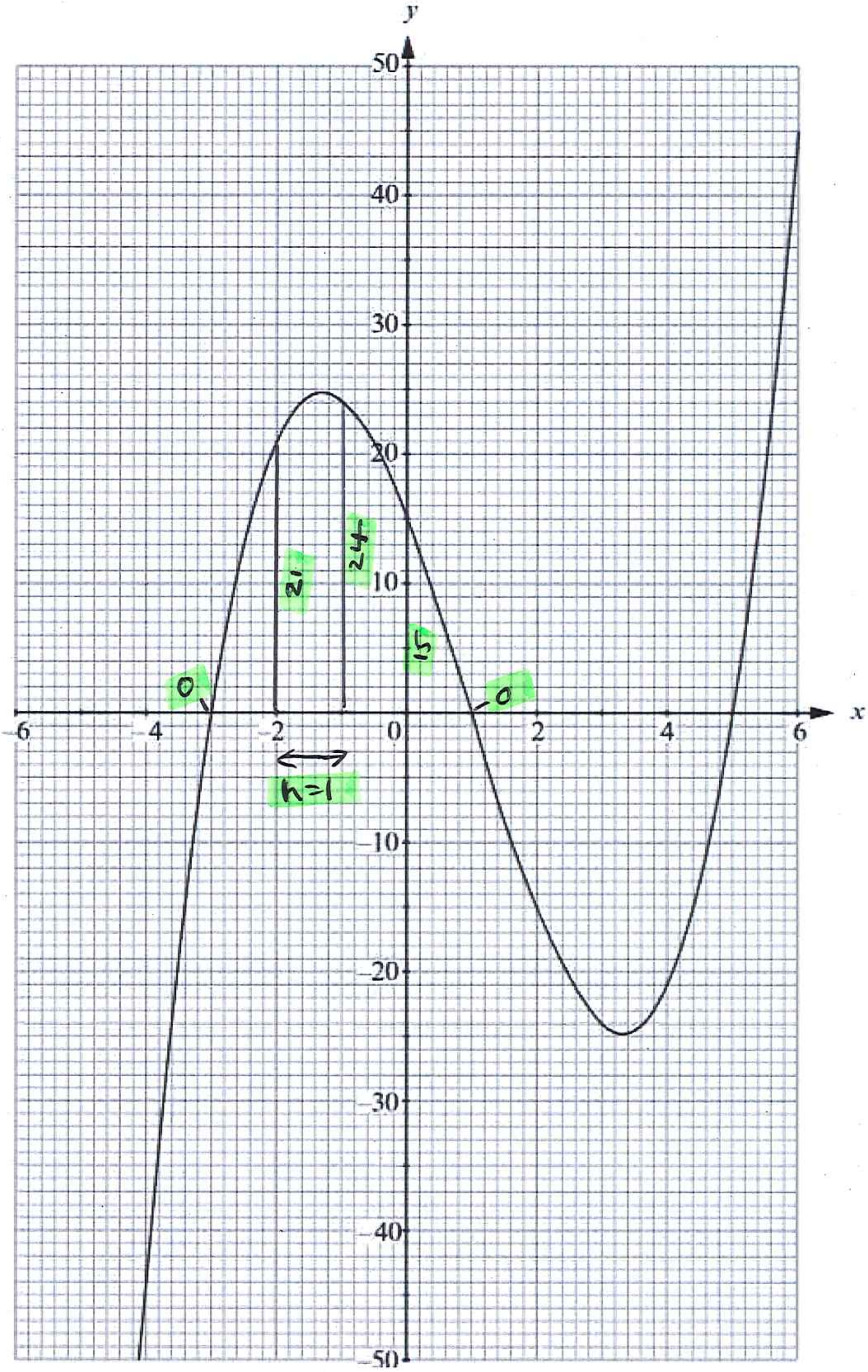
$$\underline{\underline{\text{Area}}} = \frac{h}{2}\left\{ y_0 + 2(y_1 + y_2) + y_3 \right\}$$

$$= \frac{1}{2}\left\{ 0 + 2(10 + 8) + 0 \right\}$$

$$= \underline{18 \text{ (units}^2)}$$

[4]

6. The graph of $y = x^3 - 3x^2 - 13x + 15$, for values of x between $x = -4$ and $x = 6$, has been drawn below.



$y_0 = 0$
$y_1 = 21$
$y_2 = 24$
$y_3 = 15$
$y_4 = 0$

Use the trapezium rule with 4 strips to estimate the area of the region enclosed by the curve $y = x^3 - 3x^2 - 13x + 15$ and the x-axis between $x = -3$ and $x = 1$.

$$\text{Area} = \frac{h}{2}\left\{y_0 + 2(y_1 + y_2 + y_3) + y_4\right\}$$

$$= \frac{1}{2}\left(0 + 2(21 + 24 + 15) + 0\right)$$

$$= 60 \text{ (units}^2\text{)}$$

[4]