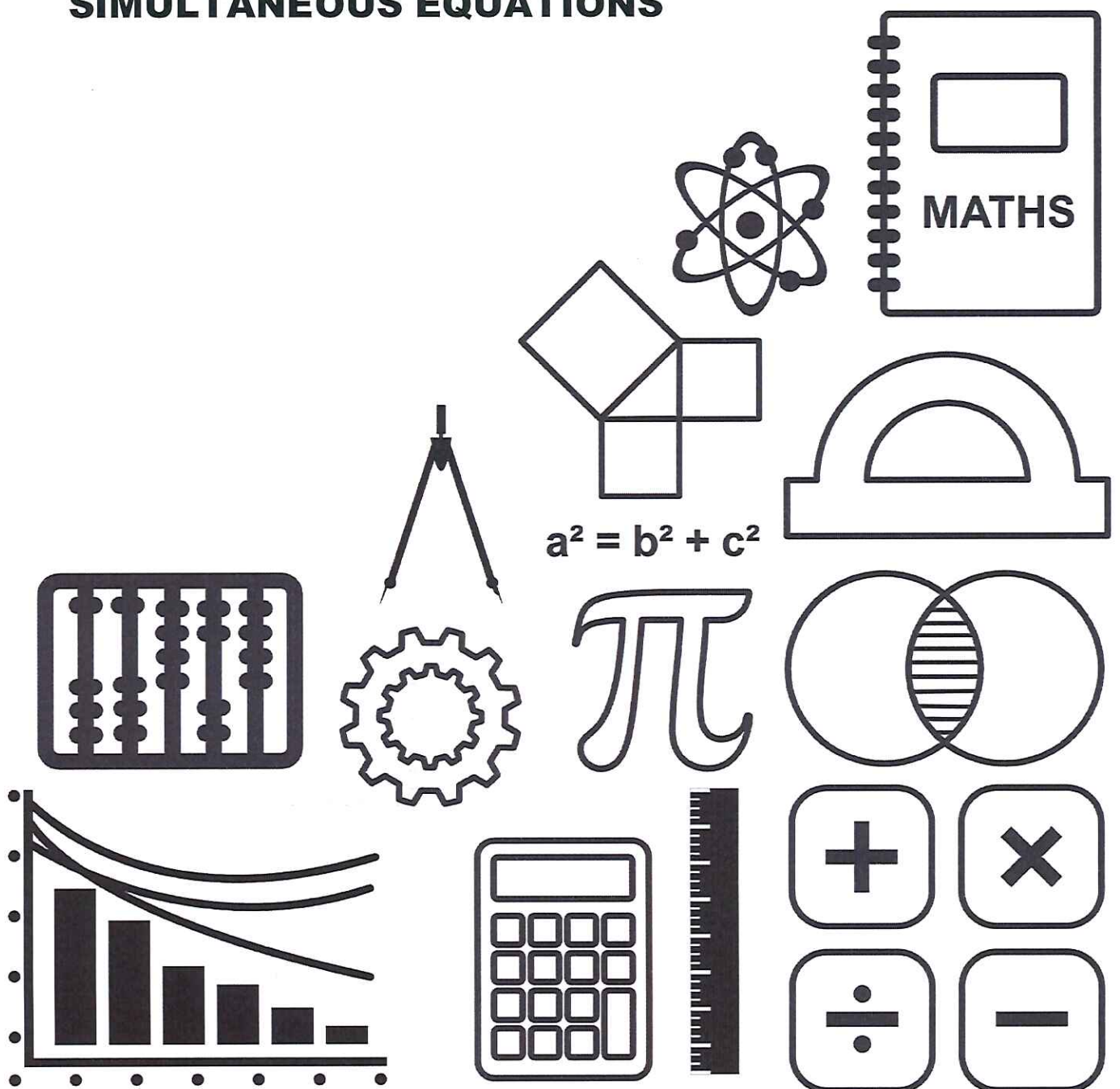


SOLUTIONS

GCSE TOPIC BOOKLET SIMULTANEOUS EQUATIONS



1.

Use an algebraic method to solve the following simultaneous equations.

[4]

$$\begin{aligned} 5a + 7b &= 63 & \textcircled{1} \\ 10a - 4b &= 18 & \textcircled{2} \end{aligned}$$

$$2 \times \textcircled{1} \quad 10a + 14b = 126$$

$$\textcircled{2} \quad 10a - 4b = 18 \quad -$$

$$18b = 108$$

$$b = \frac{108}{18}$$

$$\underline{\underline{b = 6}}$$

$$\text{In } \textcircled{1} \quad 5a + 7b = 63$$

$$5a + 7(6) = 63$$

$$5a + 42 = 63$$

$$5a = 63 - 42$$

$$5a = 21$$

$$a = \frac{21}{5}$$

$$\underline{\underline{a = 4.2}}$$

$$(\text{check in } \textcircled{2} \quad 10(4.2) - 4(6) = 18 \checkmark)$$

$$\underline{\underline{a = 4.2, b = 6}}$$

2.

Solve the following simultaneous equations using an algebraic method.
You must show all your working.

[4]

$$3x + 5y = 6.5 \quad (1)$$

$$2x - 2y = -9 \quad (2)$$

$$2 \times (1) \quad 6x + 10y = 13$$

$$3 \times (2) \quad 6x - 6y = -27 \quad -$$

$$16y = 40$$

$$y = 40/16$$

$$y = 2.5$$

$$\text{in } (1) \quad 3x + 5(2.5) = 6.5$$

$$3x = 6.5 - 12.5$$

$$3x = -6$$

$$x = -6/3$$

$$x = -2$$

$$(\text{check in } (2) \quad 2(-2) - 2(2.5) = -9) \checkmark$$

$$x = -2, y = 2.5$$

3.

Solve the following simultaneous equations using an algebraic (not graphical) method.
You must show all your working.

[4]

$$4x - 3y = 11 \quad (1)$$

$$6x - 2y = 9 \quad (2)$$

$$2 \times (1) \quad 8x - 6y = 22$$

$$3 \times (2) \quad 18x - 6y = 27 \quad -$$

$$-10x = -5$$

$$x = -5/-10$$

$$x = 0.5$$

$$\text{in } (1)$$

$$4(0.5) - 3y = 11$$

$$2 - 3y = 11$$

$$-3y = 9$$

$$y = 9/-3$$

$$y = -3$$

$$(\text{check in } (2) \quad 6(0.5) - 2(-3) = 9) \checkmark$$

$$x = 0.5, y = -3$$

4.

Use an algebraic method to solve the following simultaneous equations.

[5]

$$3x + 2y = 1 \quad \text{and} \quad 2x = 5(y - 10)$$

①

②

rearrange ②

$$2x = 5y - 50$$

$$2x - 5y = -50 \quad \text{③}$$

$$3x + 2y = 1 \quad \text{①}$$

3 × ③

$$6x - 15y = -150$$

2 × ①

$$6x + 4y = 2$$

$$-19y = -152$$

$$y = \frac{-152}{-19}$$

$$y = 8$$

in ①

$$3x + 2(8) = 1$$

$$3x + 16 = 1$$

$$3x = 1 - 16$$

$$3x = -15$$

$$x = \frac{-15}{3}$$

$$x = -5$$

(check in ②)

$$2(-5) = 5(8 - 10) \quad \checkmark$$

$$-10 = -10$$

$$x = -5, y = 8$$

5.

Glyn employs two people, Ben and Ceri.
Ben and Ceri are paid at different hourly rates.

Glyn has recorded how many hours Ben and Ceri have worked on Monday and Tuesday.
He has also recorded the total amount he has paid out in wages.

Day	Number of hours worked		Total pay (£)
	Ben	Ceri	
Monday	6	5	116
Tuesday	4	8	138

b = rate Ben is paid

c = rate Ceri is paid

Use an algebraic method to calculate how much Ben and Ceri are paid per hour.

[6]

$$6b + 5c = 116 \quad (1)$$

$$4b + 8c = 138 \quad (2)$$

$$2 \times (1) \quad 12b + 10c = 232$$

$$3 \times (2) \quad 12b + 24c = 414 \quad -$$

$$-14c = -182$$

$$c = \frac{-182}{-14}$$

$$c = 13 = \pounds 13.00$$

$$\text{in } (1) \quad 6b - 5(13) = 116$$

$$6b + 65 = 116$$

$$6b = 116 - 65$$

$$6b = 51$$

$$b = 51/6 \quad b = 8.5 = \pounds 8.50$$

$$(\text{check in } (2) \quad 4(8.5) + 8(13) = 138) \checkmark$$

Ben is paid £ 8.50 per hour

Ceri is paid £ 13.00 per hour

6.

Tomos, Gwen and Jen visit the same fruit stall.

Tomos buys 4kg of raspberries and 5kg of blackcurrants for a total cost of £38.

Gwen buys 6kg of raspberries and 3kg of blackcurrants for a total cost of £39.

Use an algebraic method to find how much Jen pays in total for 5kg of raspberries and 7kg of blackcurrants. [6]

Let r = price of 1kg of raspberries

& b = price of 1kg of blackcurrants

$$4r + 5b = 38 \quad (1)$$

$$6r + 3b = 39 \quad (2)$$

$$3 \times (1) \quad 12r + 15b = 114$$

$$2 \times (2) \quad 12r + 6b = 78 \quad -$$

$$9b = 36$$

$$b = 36/9 \quad \underline{b = 4}$$

in (1) $4r + 5(4) = 38$

$$4r + 20 = 38$$

$$4r = 38 - 20$$

$$4r = 18$$

$$r = 18/4 \quad \underline{r = 4.5}$$

(check in (2) $6(4.5) + 3(4) = 39 \checkmark$)

Raspberries cost

£4.50/kg

Blackcurrants cost

£4/kg

Jen buys 5kg raspberries & 7kg blackcurrants.

This costs $5 \times (4.50) + 7 \times (4)$

$$= \underline{\underline{£50.50}}$$

7.

Sumston Village School organised a concert to raise money for charity.

All of the 120 tickets were sold for a total of £1210.

The price of an adult's ticket was £12.

The price of a child's ticket was £7.

a = number of adult tickets

c = number of child tickets

How many adult tickets and how many child tickets were sold?

You must use an algebraic method.

[4]

$$a + c = 120 \quad (1)$$

$$12a + 7c = 1210 \quad (2)$$

$$7 \times (1) \quad 7a + 7c = 840$$

$$(2) \quad 12a + 7c = 1210 \quad -$$

$$-5a = -370$$

$$a = -370 / -5$$

$$a = 74$$

$$\text{in } (1) \quad a + c = 120$$

$$74 + c = 120$$

$$c = 120 - 74$$

$$c = 46$$

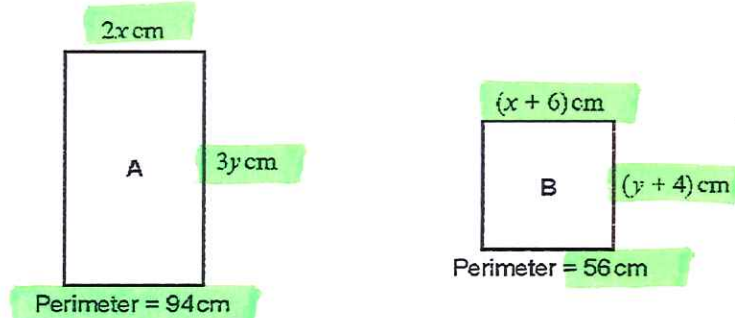
$$(\text{check in } (2) \quad 12(74) + 7(46) = 1210 \checkmark)$$

Number of adult tickets = 74

Number of child tickets = 46

8.

The rectangles A and B have perimeters of 94cm and 56cm as shown below.



Diagrams not drawn to scale

Use an algebraic method to calculate the area of each rectangle.

[8]

Shape A gives $4x + 6y = 94$ ①

Shape B gives $2(x + 6) + 2(y + 4) = 56$

$$2x + 2y + 12 + 8 = 56$$

$$2x + 2y = 36$$
 ②

① $4x + 6y = 94$

$2 \times$ ② $4x + 4y = 72$ —

$$2y = 22$$

$$y = 22/2 \quad \underline{y = 11}$$

in ① $4x + 6(11) = 94$

$$4x = 94 - 66$$

$$4x = 28 \quad x = 28/4 \quad \underline{x = 7}$$

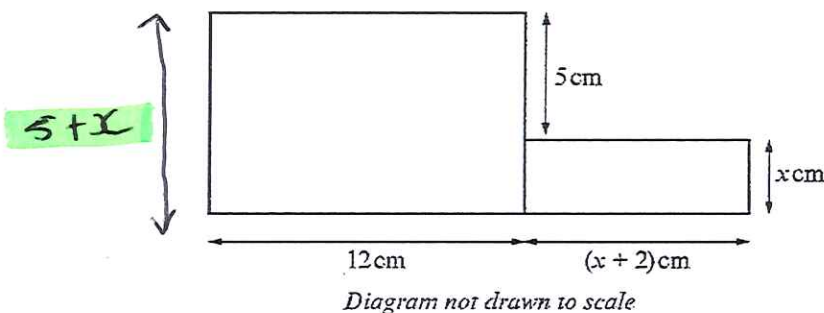
(check in ② $2(7) + 2(11) = 36$) ✓

Area of A = $L \times W = 2x \times 3y$
 $= 14 \times 33 = \underline{462 \text{ cm}^2}$

Area of B = $L \times W = (x + 6) \times (y + 4)$
 $= (7 + 6) \times (11 + 4)$
 $= 13 \times 15 = \underline{195 \text{ cm}^2}$

9.

The diagram below shows a composite shape formed by joining two rectangles.



The area of the larger rectangle is $4y \text{ cm}^2$.
The area of the smaller rectangle is $y \text{ cm}^2$.

Form and solve simultaneous equations to calculate the dimensions of the smaller rectangle.
Give your answers correct to 1 decimal place.

Smaller rectangle gives $x(x+2) = y$ (1)

Larger rectangle gives $12(5+x) = 4y$ (2) because $x(x+2) = y$

Substitute

(1) into (2)

$$12(5+x) = 4x(x+2)$$

$$60 + 12x = 4x^2 + 8x$$

$$0 = 4x^2 - 4x - 60 \quad \div 4$$

$$0 = x^2 - x - 15$$

Tip In the question we are told to give answers to 1dp
this is a massive clue that it won't factorise. So use the
quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$a=1 \quad b=-1 \quad c=-15$$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-15)}}{2(1)}$$

$$x = \frac{1 \pm \sqrt{61}}{2}$$

$$x = -3.4 \text{ (discard)} \quad [8]$$

$$\underline{\underline{x = 4.4 \text{ cm}}}$$

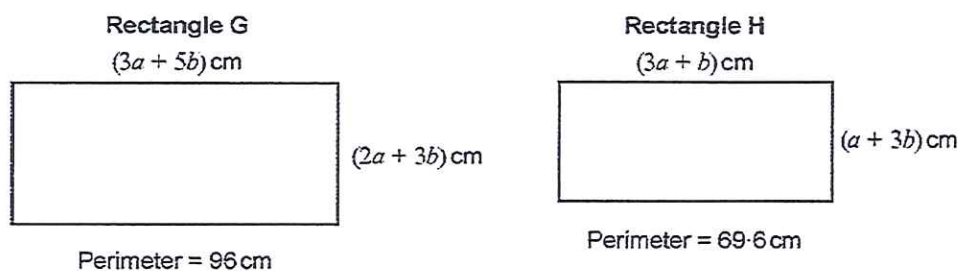
$\therefore$ Dimensions of the smaller rectangle are

$$\underline{\underline{4.4 \text{ cm}}} \text{ and } \underline{\underline{6.4 \text{ cm}}}$$

Notice you don't actually
need to find y !

10.

The dimensions and perimeters of two rectangles, G and H, are shown below.



Diagrams not drawn to scale

Freya has written an equation for rectangle G:

$$10a + 16b = 96 \quad (1)$$

(a) Write down an equation for rectangle H.

$$(3a + b) \times 2 + (a + 3b) \times 2 = 69.6 \quad (2) \quad [1]$$

$$\underline{8a + 8b = 69.6} \quad (\text{or } a + b = 8.7) \quad \text{if you } \div 8$$

(b) Use an algebraic method to calculate a and b .
Hence, calculate the dimensions of rectangle H.

[5]

$$\begin{array}{rcl} (1) & 10a + 16b & = 96 \\ 10 \times (2) & 10a + 10b & = 87 \quad - \\ \hline & 6b & = 9 \\ & b & = 9/6 \quad \underline{b = 1.5} \end{array}$$

$$\begin{array}{rcl} \text{in } (2) & a + b & = 8.7 \\ & a + 1.5 & = 8.7 \quad \underline{a = 7.2} \end{array}$$

Dimensions of rectangle H:-

$$\begin{aligned} \text{width} &= (a + 3b) = (7.2 + 3(1.5)) \\ \text{width} &= 11.7 \text{ cm} \\ \text{length} &= (3a + b) = (3(7.2) + 1.5) \\ \text{length} &= 23.1 \text{ cm} \end{aligned}$$

Length of rectangle H is $\underline{23.1}$ cm

Width of rectangle H is $\underline{11.7}$ cm

11.

Let $b = \text{price of 1 bench}$ £
 $c = \text{price of 1 chair}$

The Davies family want to buy some garden furniture. Their local garden centre stocks the particular brand they would like.

The price of one garden bench and four chairs is £310.

The price of two garden benches and three chairs is £345.

$$b + 4c = 310 \quad (1)$$

$$2b + 3c = 345 \quad (2)$$

The Davies family have £450 available to spend.

Do they have enough money to buy two garden benches and five chairs?

You must show all your working.

[5]

$$2 \times (1) \quad 2b + 8c = 620$$

$$(2) \quad 2b + 3c = 345 \quad -$$

$$5c = 275$$

$$c = 275/5$$

$$c = \underline{\underline{£55}}$$

$$\text{in } (1) \quad b + 4(55) = 310$$

$$b + 220 = 310$$

$$b = 310 - 220$$

$$b = \underline{\underline{£90}}$$

$$(\text{check in } (2) \quad 2(90) + 3(55) = 345) \quad \checkmark$$

Check if the Davies family can afford

$$2b + 5c = 2(90) + 5(55)$$

$$= \underline{\underline{£455}}$$

No. The Davies family don't have enough money.

12.

Simon and Syra are on holiday in Devon.
They buy some holiday souvenirs for their friends.
Simon pays £2.05 for 2 key rings and 3 pencils.
Syra pays £3.20 for 3 key rings and 5 pencils.
All the key rings are the same price and all the pencils are the same price.

k = price of 1 keyring

p = price of 1 pencil

Find the individual prices of a key ring and a pencil.
You must use an algebraic method.

TIP change costs to pence^[5]
it will make your life easier

$$2k + 3p = 205 \quad (1)$$

$$3k + 5p = 320 \quad (2)$$

$$3 \times (1) \quad 6k + 9p = 615$$

$$2 \times (2) \quad 6k + 10p = 640 \quad -$$

$$-p = -25$$

$$p = 25 \text{ pence}$$

in (1) $2k + 3(25) = 205$

$$2k + 75 = 205$$

$$2k = 205 - 75$$

$$2k = 130, \quad k = 130/2, \quad k = 65 \text{ pence}$$

(check in (2) $3(65) + 5(25) = 320 \checkmark$)

Price of a key ring = 65 pence (€0.65)

Price of a pencil = 25 pence (€0.25)