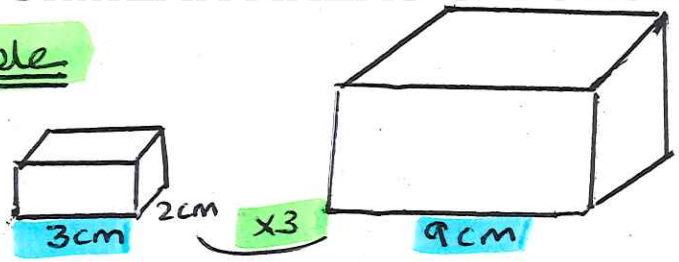
MATHSDIY

SOLUTIONS

GCSE TOPIC BOOKLET
SIMILAR AREAS & VOLUMES

Example

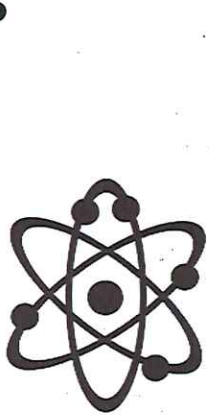


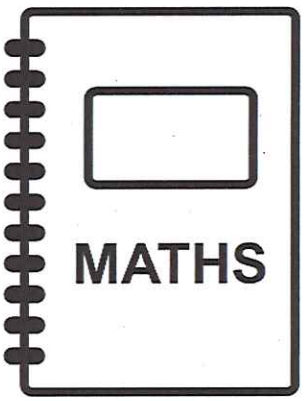
If the cuboids are similar

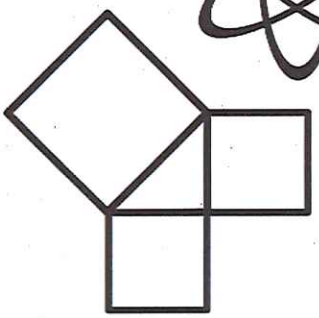
Length scale Factor = 3

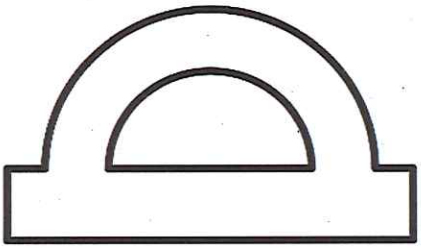
Area scale Factor
= (Length Scale Factor)^2 = 3^2 = 9

Volume Scale Factor
= (Length Scale Factor)^3
= 3^3 = 27

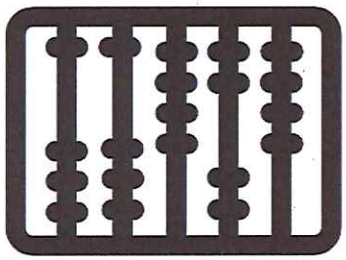


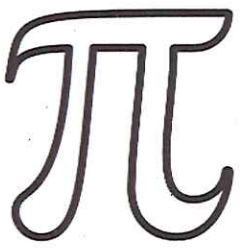


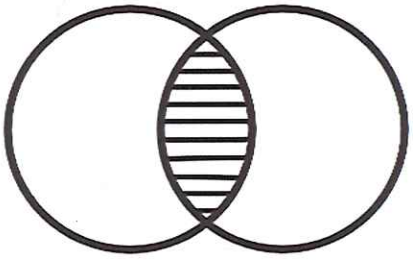


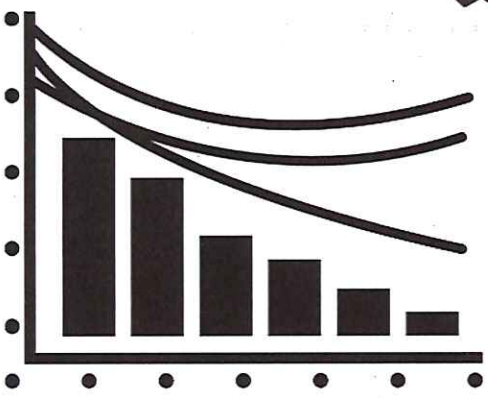


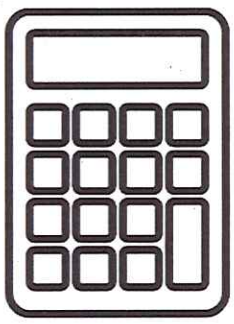
$a^2 = b^2 + c^2$

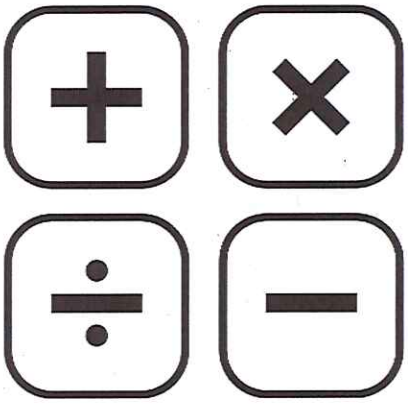




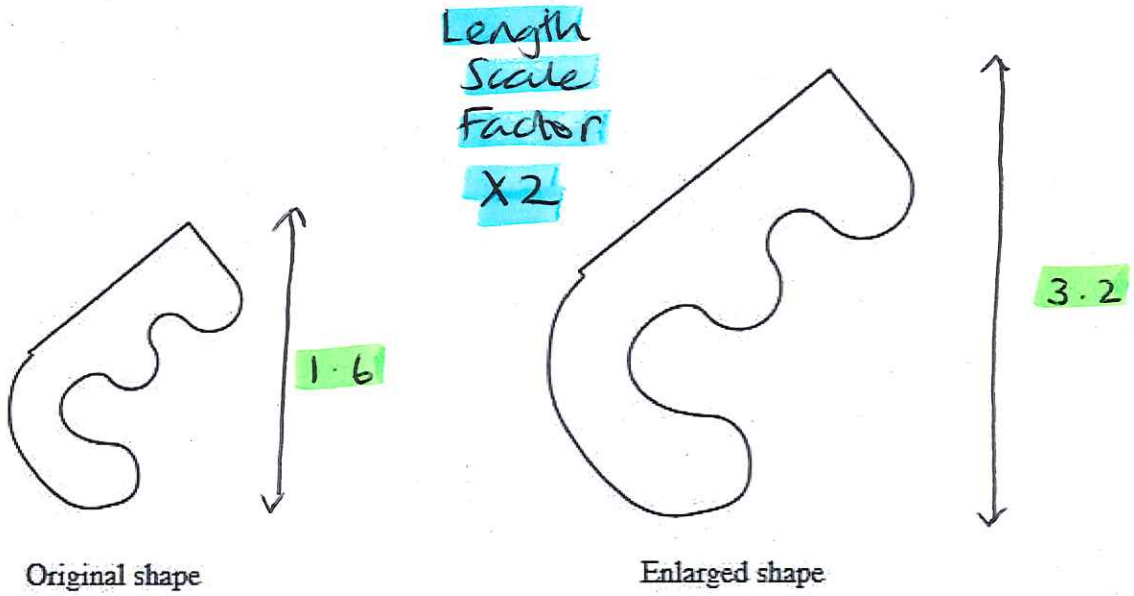








1. The diagram shows a shape and an enlargement of the shape.



Original shape

Enlarged shape

Diagram not drawn to scale.

The height of the original shape is 1·6 cm and the height of the enlarged shape is 3·2 cm. The area of the original shape is 5·2 cm^2. Find the area of the enlarged shape.

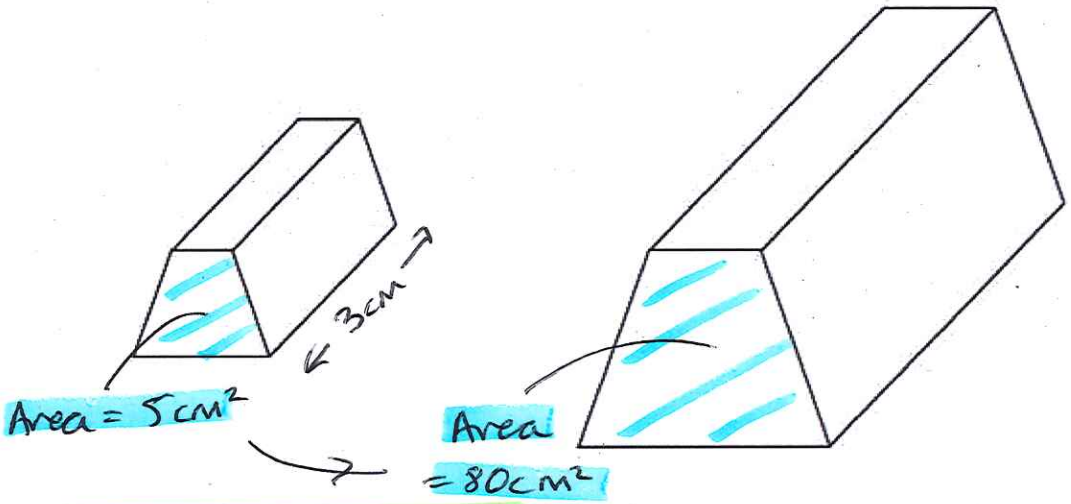
L·S·F = 2

A·S·F = 2^2 = 4

Enlarged shape's area = 4 × 5·2

= 20·8 cm^2 [2]

2. The prisms shown below are similar.



Diagrams not drawn to scale.

Area Scale Factor = 80 ÷ 5
= 16

The area of the uniform cross-section of the smaller prism is 5 cm^2 and its length is 3 cm. The area of the cross-section of the larger prism is 80 cm^2. Calculate the volume of the larger prism.

Area Scale Factor = 80 ÷ 5 = 16

∴ Length Scale Factor = $\sqrt{16}$ = 4

∴ Volume Scale Factor = (Length scale factor)^3

= 4^3 = 64

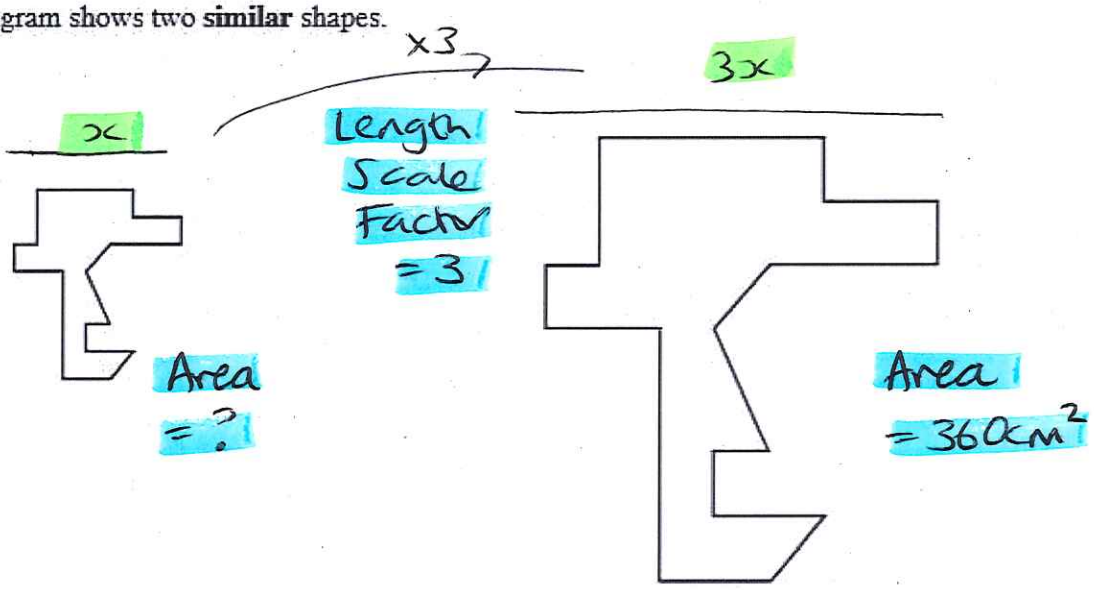
Volume of small prism = 5 × 3 = 15cm^3

Volume of larger prism = 15 × 64

= 960cm^3

[5]

3. The diagram shows two **similar** shapes.



Diagrams not drawn to scale.

Each length on the larger shape is three times the corresponding length on the smaller shape.
The area of the larger shape is 360 cm^2. Find the area of the smaller shape.

Length scale Factor = 3

Area scale Factor = $3^2 = 9$

Area of Smaller Shape = $360 \div 9$

= 40cm^2

[3]

4. The diagram shows two **similar** shapes.

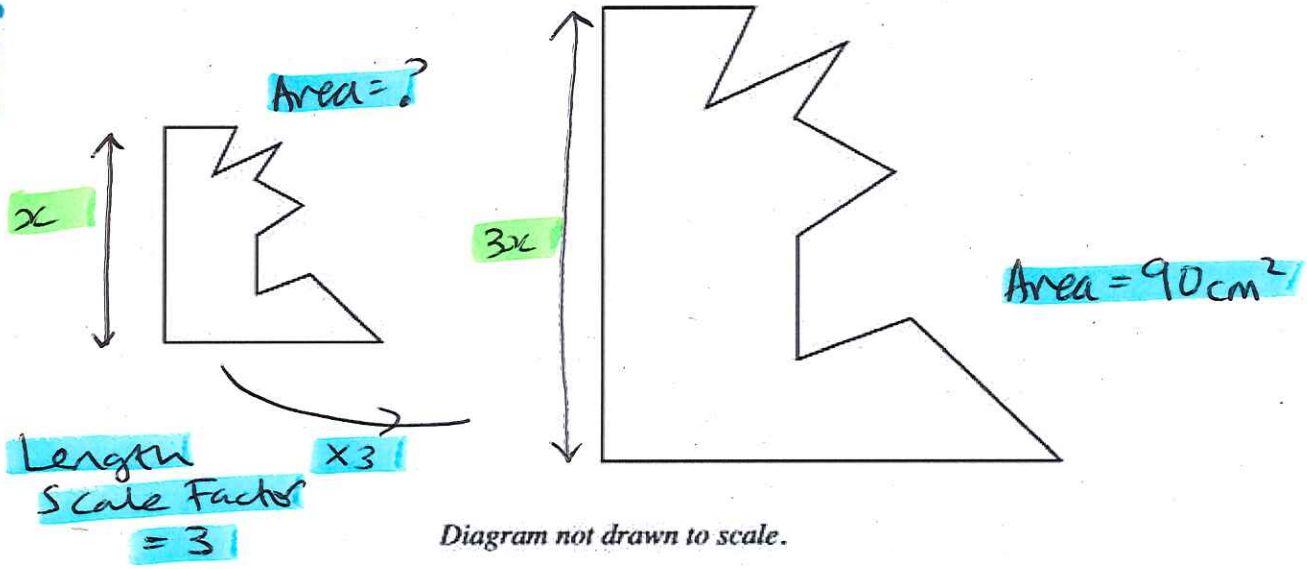


Diagram not drawn to scale.

Each length on the larger shape is three times the corresponding length on the smaller shape.
The area of the larger shape is 90 cm^2. Find the area of the smaller shape.

Length Scale Factor = 3

Area Scale Factor = $3^2 = 9$

Area of Smaller Shape = $90 \div 9 = 10$ cm^2

[3]

5. The diagram shows two similar shapes.

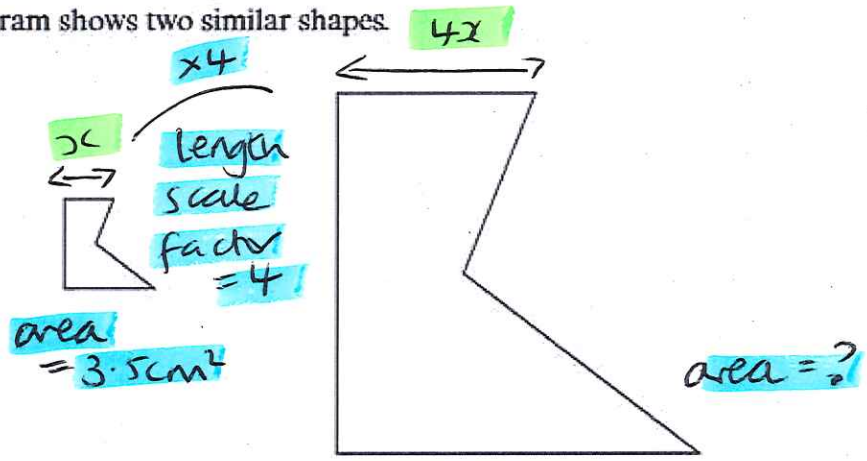


Diagram not drawn to scale

Each length on the larger shape is four times the corresponding length on the smaller shape. The area of the smaller shape is $3{\cdot}5\,cm^2$. Find the area of the larger shape.

Length Scale Factor = 4

Area Scale Factor = $(4)^2 = 16$

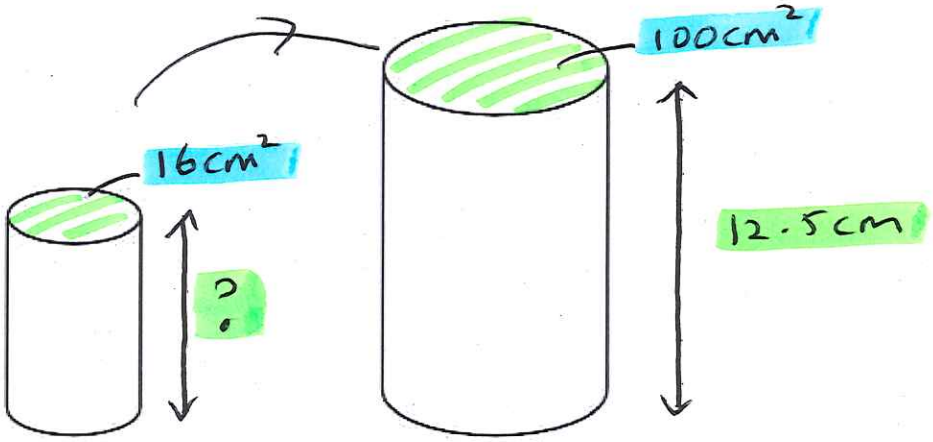
Area of large shape = $3{\cdot}5 \times 16$

Area of larger shape 56 cm^2

[3]

6. The diagram shows two similar cylinders.

Area Scale Factor $= \frac{100}{16} = 6.25$



Diagrams not drawn to scale.

The areas of the ends of the smaller and larger cylinders are 16 cm^2 and 100 cm^2 respectively. Given that the height of the larger cylinder is 12·5 cm, find the height of the smaller cylinder.

Area scale Factor $= 100/16 = 6.25$

Length scale Factor $= \sqrt{6.25} = 2.5$

Smaller Cylinder Height $= 12.5 \div 2.5$

$= 5\text{cm}$

[3]

7. The diagram shows two shapes.

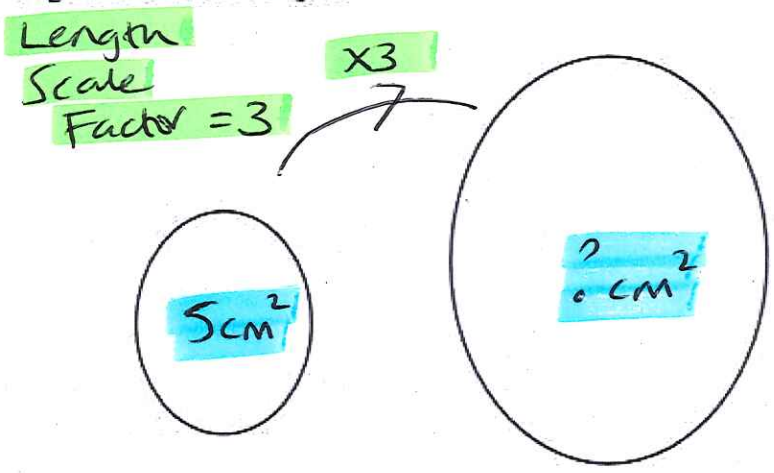


Diagram not drawn to scale.

The larger shape is an enlargement of the smaller shape with a scale factor 3. The area of the smaller shape is 5cm^2. Calculate the area of the larger shape.

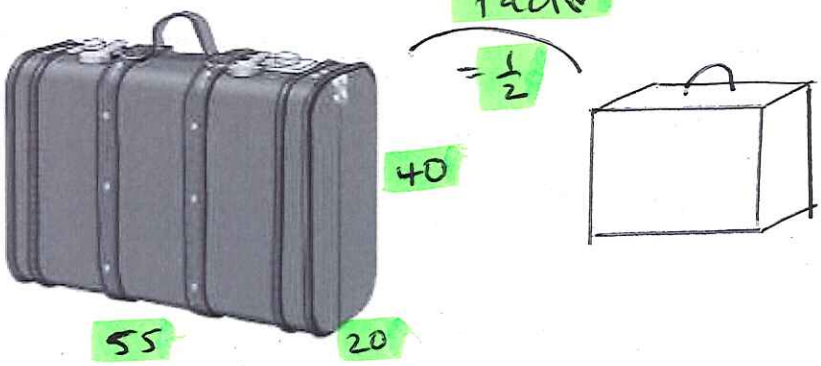
Length scale Factor = 3

Area scale Factor = $3^2 = 9$

Area of Large Shape = $5 \times 9 = 45\text{cm}^2$

[3]

8. A company produces suitcases with dimensions 55 cm by 40 cm by 20 cm.



The company has been asked to produce a new, smaller suitcase.

- The new suitcase must be **similar** to the original suitcase, with dimensions in the same proportions.
- The volume of the new smaller suitcase should be **half** the volume of the original suitcase.

Find the dimensions of the new smaller suitcase.
Give your dimensions correct to an appropriate degree of accuracy. [4]

Volume Scale Factor = $\frac{1}{2}$

Length Scale Factor = $\sqrt[3]{\frac{1}{2}}$ = 0.7937 (4dp)

New length = 0.7937 × 55cm = 43.7 cm (1dp)
New width = 0.7937 × 40 cm = 31.7 cm (1dp)
New height = 0.7937 × 20 cm = 15.9 cm (1dp)

9.

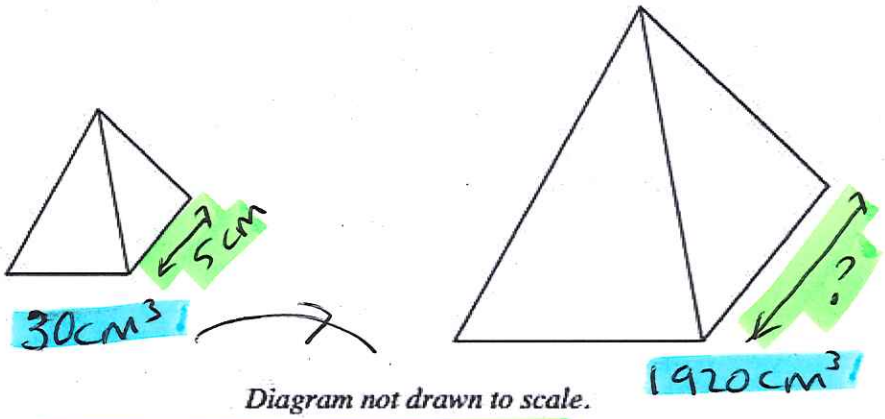


Diagram not drawn to scale.

Volume scale factor = 64

The diagram shows two similar square based pyramids, the volume of the smaller pyramid is $30\,cm^3$ and the volume of the larger pyramid is $1920\,cm^3$.
The length of the side of the base of the smaller square based pyramid is 5 cm.
Calculate the length of the side of the base of the larger pyramid.

Volume Scale Factor = 64

Length Scale Factor = $\sqrt[3]{64} = 4$

Length of side of base of larger pyramid
$= 4 \times 5cm = 20cm$

[3]