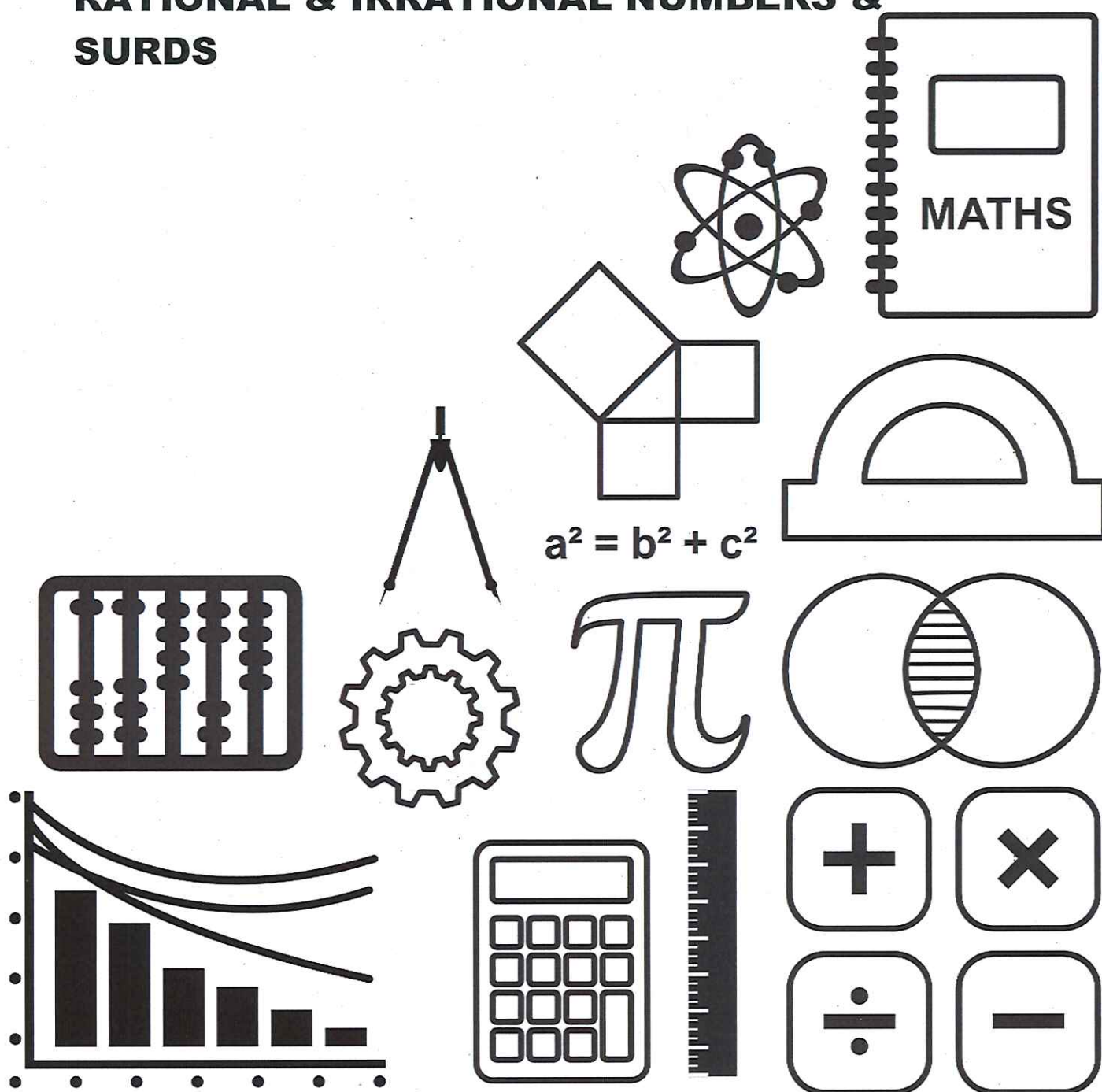


MATHSDIY

GCSE TOPIC BOOKLET

SOLUTIONS

RATIONAL & IRRATIONAL NUMBERS &
SURDS



1. Expand $(5 - \sqrt{2})^2$ and state whether the result is rational or irrational.

$$\begin{aligned}
 &= (5 - \sqrt{2})(5 - \sqrt{2}) \\
 &= 25 - 5\sqrt{2} - 5\sqrt{2} + 2 \\
 &= \underline{27 - 10\sqrt{2}} \quad \underline{\text{irrational}}
 \end{aligned}$$

(2)

2. Simplify $(\sqrt{75} - \sqrt{3})^2$ and state whether your answer is rational or irrational.

$$\begin{aligned}
 &= (\sqrt{75} - \sqrt{3})(\sqrt{75} - \sqrt{3}) \\
 &= 75 - \sqrt{3}\sqrt{75} - \sqrt{3}\sqrt{75} + 3 \\
 &= 78 - 2\sqrt{225} \\
 &= 78 - 2 \times 15 = \underline{48} \quad \underline{\text{rational}}
 \end{aligned}$$

(2)

3. Show that $(\sqrt{72} - \sqrt{2})^2 = 50$.

$$\begin{aligned}
 &(\sqrt{72} - \sqrt{2})(\sqrt{72} - \sqrt{2}) \\
 &= 72 - \sqrt{2}\sqrt{72} - \sqrt{2}\sqrt{72} + 2 \\
 &= 74 - \sqrt{144} - \sqrt{144} \\
 &= 74 - 12 - 12 \\
 &= \underline{50}
 \end{aligned}$$

(2)

4. Write down a value of x for which $x^{\frac{3}{2}}$ is rational.

examples could be

<u>1</u>	$1^{\frac{3}{2}} = 1$
<u>0</u>	$0^{\frac{3}{2}} = 0$
<u>4</u>	$4^{\frac{3}{2}} = 2^3 = 8$

(2)

5. Show that $(\sqrt{3} + \sqrt{12})^2 = 27$.

$$= (\sqrt{3} + \sqrt{4} \sqrt{3})^2$$

$$= (\sqrt{3} + 2\sqrt{3})^2$$

$$= (3\sqrt{3})^2$$

$$= 3\sqrt{3} \times 3\sqrt{3}$$

$$= 9 \times 3 = \underline{\underline{27}}$$

(2)

6. Give an example of an irrational number

- a) whose square is rational,

eg $\sqrt{2}$ is irrational and $(\sqrt{2})^2 = 2$
is rational

(1)

- b) whose square is irrational.

eg π is irrational and π^2 is irrational

(2)

7. Find the value of $(\sqrt{32} + \sqrt{2})^2$

$$(\sqrt{32} + \sqrt{2})^2$$

$$= (\sqrt{16} \sqrt{2} + \sqrt{2})^2$$

$$= (4\sqrt{2} + \sqrt{2})^2$$

$$= (5\sqrt{2})^2 = 5\sqrt{2} \times 5\sqrt{2}$$

$$= 25 \times 2 = \underline{\underline{50}}$$

(2)

8. Given that $x = \sqrt{12}$, $y = \sqrt{3}$ and $z = \sqrt{6}$, simplify each of the following, indicating in each case whether your answer is rational or irrational.

a) $xy - 4$

$$= \sqrt{12} \times \sqrt{3} - 4 = \sqrt{36} - 4 = 6 - 4 = 2$$

rational

b) $\frac{x}{yz^2} = \frac{\sqrt{12}}{\sqrt{3} \times (\sqrt{6})^2} = \frac{\sqrt{4} \sqrt{3}}{\sqrt{3} \times 6} = \frac{2}{6} = \frac{1}{3}$

rational

c) $(z + y)^2 = (\sqrt{3} + \sqrt{6})^2$
 $= (\sqrt{3} + \sqrt{6})(\sqrt{3} + \sqrt{6})$
 $= 3 + \sqrt{18} + \sqrt{18} + 6 = 9 + 2\sqrt{18}$
 $= 9 + 2\sqrt{9 \times 2} = 9 + 6\sqrt{2}$

irrational

(5)

9. Do not use a calculator when answering this question. All working must be shown. Simplify each of the following, indicating in each case whether your answer is rational or irrational.

a) $\frac{6}{\sqrt{3}} + \sqrt{48} = \frac{6\sqrt{3}}{3} + \sqrt{16} \sqrt{3}$

$$= 2\sqrt{3} + 4\sqrt{3}$$

$$= 6\sqrt{3}$$

irrational

b) $(3 - \sqrt{2})^2 = (3 - \sqrt{2})(3 - \sqrt{2})$
 $= 9 - 3\sqrt{2} - 3\sqrt{2} + 2$

$$= 11 - 6\sqrt{2}$$

irrational

(4)

10. Evaluate $\frac{(7-\sqrt{3})(7+\sqrt{3})}{2}$. $= \frac{49 - 7\sqrt{3} + 7\sqrt{3} - 3}{2}$

State clearly whether your answer is rational or irrational.

$$= \frac{49-3}{2} = \frac{46}{2} = \underline{\underline{23}}$$

rational

(2)

11. Simplify $\frac{(5\sqrt{3})^2 - \frac{2\sqrt{18}}{\sqrt{2}}}{\sqrt{32} \times \sqrt{2}}$. $= \frac{5\sqrt{3} \times 5\sqrt{3} - 2\sqrt{9} \sqrt{2} \div \sqrt{2}}{\sqrt{2} \sqrt{16} \sqrt{2}}$

State clearly whether your answer is rational or irrational.

$$= \frac{(25 \times 3) - (2 \times 3)}{(2 \times 4)} = \frac{69}{8} = \underline{\underline{8\frac{5}{8}}}$$

8.625 rational

(5)

12. Simplify $(\pi\sqrt{20} - \pi\sqrt{5})^2$, leaving your answer in terms of π .

$$\begin{aligned} &= (\pi\sqrt{20} - \pi\sqrt{5})(\pi\sqrt{20} - \pi\sqrt{5}) \\ &= 20\pi^2 - \pi^2\sqrt{100} - \pi^2\sqrt{100} + 5\pi^2 \\ &= 25\pi^2 - 20\pi^2 \end{aligned}$$

$$= \underline{\underline{5\pi^2}}$$

(3)

13. Given that $f = \sqrt{2}$, $g = \sqrt{5}$ and $h = \sqrt{10}$, simplify each of the following, indicating in each case whether your answer is rational or irrational.

a) $fg + h = \sqrt{2}\sqrt{5} + \sqrt{10}$
 $= \sqrt{10} + \sqrt{10} = \underline{2\sqrt{10}}$ irrational

b) $\frac{fg}{h} = \frac{\sqrt{2} \times \sqrt{5}}{\sqrt{10}} = \frac{\sqrt{10}}{\sqrt{10}} = \underline{1}$ rational

c) $fh = \sqrt{2} \times \sqrt{10} = \sqrt{20} = \sqrt{4 \times 5}$
 $= \underline{2\sqrt{5}}$ irrational

(3)

14. Simplify $\sqrt{288}$, leaving your answer in surd form.

$= \sqrt{2 \times 144} = \underline{12\sqrt{2}}$

(2)

15. Simplify $\sqrt{3}(5 + \sqrt{3}) - \sqrt{3}(5 - 2\sqrt{3})$.

$= \cancel{5\sqrt{3}} + 3 - \cancel{5\sqrt{3}} + 2\sqrt{3}\sqrt{3} = 3 + 2(3)$
 $= \underline{9}$

(2)

16. Evaluate $(\sqrt{3})^6$.

$$= (\sqrt{3^2})^3$$

$$= 3^3 = 27$$

(1)

17. Simplify $(2 + 3\sqrt{2})(5 - \sqrt{2})$.

$$= 10 + 15\sqrt{2} - 2\sqrt{2} - 3(2)$$

$$= 4 + 13\sqrt{2}$$

(3)

18. Evaluate $(7\sqrt{2} - 4\sqrt{2})^4$.

$$= (3\sqrt{2})^4$$

$$= 3\sqrt{2} \times 3\sqrt{2} \times 3\sqrt{2} \times 3\sqrt{2}$$

$$= 81 \times 2 \times 2 = 324$$

(3)

19. Simplify $(3\sqrt{5} - \sqrt{2})(3\sqrt{5} + \sqrt{2})$ and state whether your answer is rational or irrational.

$$= 9(5) + \cancel{3\sqrt{5}\sqrt{2}} - \cancel{3\sqrt{5}\sqrt{2}} - \sqrt{2}\sqrt{2}$$

$$= 45 - 2$$

$$= 43$$

(3)