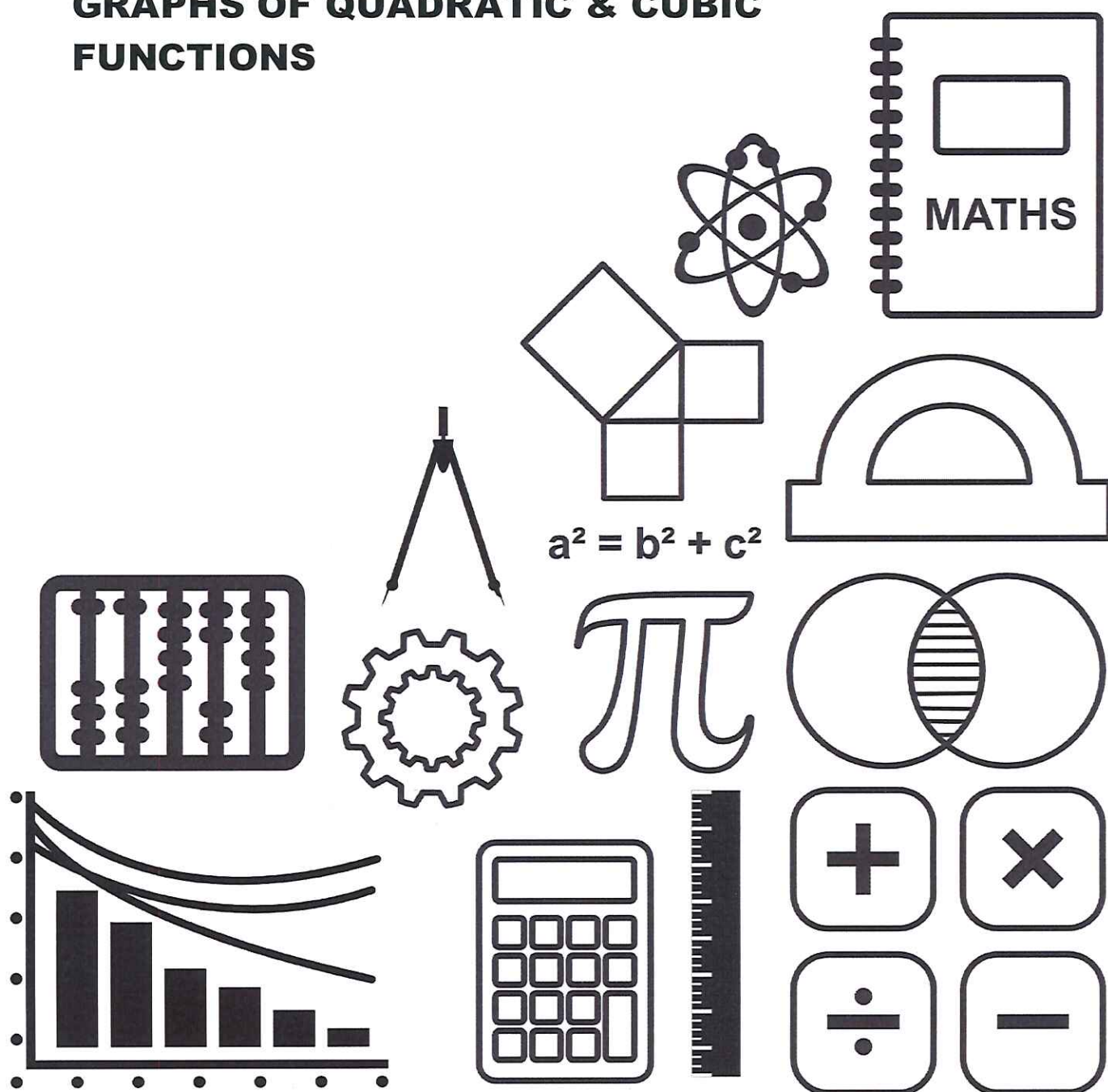


SOLUTIONS

GCSE TOPIC BOOKLET GRAPHS OF QUADRATIC & CUBIC FUNCTIONS



1.

- (a) The table below shows some of the values of $y = 2x^2 - 5x - 1$ for values of x from -2 to 4.

Complete the table by finding the value of y for $x = -1$ and for $x = 2$.

[2]

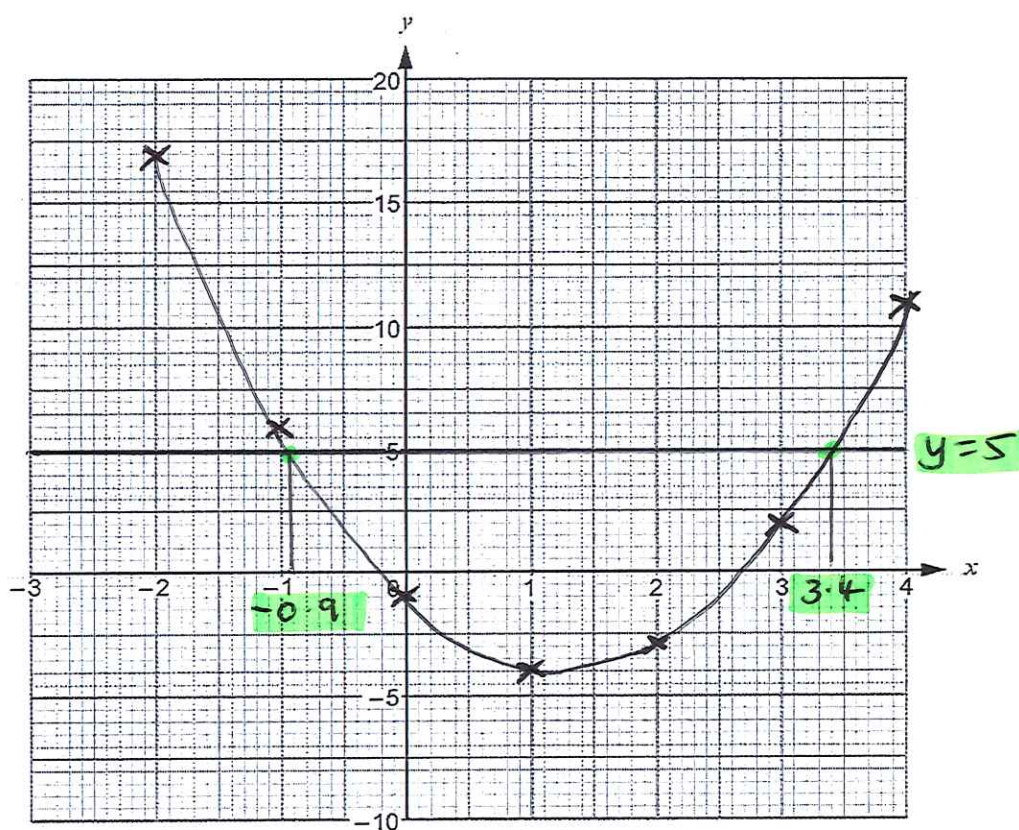
x	-2	-1	0	1	2	3	4
$y = 2x^2 - 5x - 1$	17	6	-1	-4	-3	2	11

When $x = 2$ $y = 2(2)^2 - 5(2) - 1 = -3$

When $x = -1$ $y = 2(-1)^2 - 5(-1) - 1 = 6$

- (b) On the graph paper below, draw the graph of $y = 2x^2 - 5x - 1$ for values of x from -2 to 4.

[2]



Smooth curve through
the centre of each
plotted point

- (c) Draw the line $y = 5$ on the graph paper.

Write down the values of x where the line $y = 5$ cuts the curve $y = 2x^2 - 5x - 1$.
Give your answers correct to 1 decimal place.

[2]

Values of x are

3.4

and

-0.9

- (d) Circle the equation below whose solutions are the values you have given in (c).

[1]

$$2x^2 - 5x - 1 = 0$$

$$2x^2 - 5x - 6 = 0$$

$$2x^2 - 5x - 5 = 0$$

$$2x^2 - x - 1 = 0$$

$$2x^2 - 5x + 4 = 0$$

You have actually solved graphically two simultaneous equations

$$y = 2x^2 - 5x - 1$$

$$y = 5$$

$$0 = 2x^2 - 5x - 6$$

2.

- (a) On the graph paper below, draw the graph of $y = 2x^2 - x - 3$ for values of x from -3 to 3. [4]

x	-3	-2	-1	0	1	2	3
y	18	7	0	-3	-2	3	12

$$x = -3, y = 2(-3)^2 - (-3) - 3 = 18$$

$$x = 1, y = 2(1)^2 - (1) - 3 = -2$$

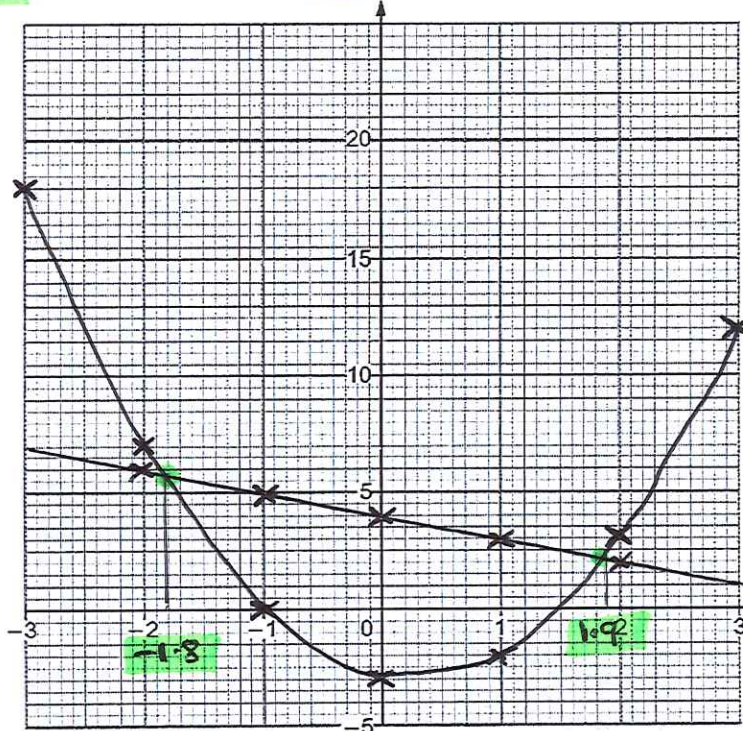
$$x = -2, y = 2(-2)^2 - (-2) - 3 = 7$$

$$x = 2, y = 2(2)^2 - (2) - 3 = 3$$

$$x = -1, y = 2(-1)^2 - (-1) - 3 = 0$$

$$x = 3, y = 2(3)^2 - (3) - 3 = 12$$

$$x = 0, y = 2(0)^2 - (0) - 3 = -3$$



$$y = -x + 4$$

Smooth curve through the centre of each plotted point.

- (b) Use your graph to write down the solutions of the equation $2x^2 - x - 3 = 0$. [1]

$$x = 1.5, x = -1$$

- (c) By drawing an appropriate straight line on the same set of axes, use your graph to solve the equation $2x^2 - 7 = 0$. [3]

$$\text{drawn } y = 2x^2 - x - 3$$

$$- y = -x + 4$$

$$0 = 2x^2 - 7$$

because we want to end up with $0 = 2x^2 - 7$ the line we need to draw is $y = -x + 4$

x	-2	-1	0	1	2
y	6	5	4	3	2

$$x = -1.8, x = 1.9$$

* These are my solutions from my curve. The examiner will check your curve. Your values might differ from mine.

3.

The table shows some of the values of $y = 3x^2 + x + 2$ for values of x from -2 to 3 .

(a) Complete the table by finding the value of y for $x = -1$ and $x = 2$.

[2]

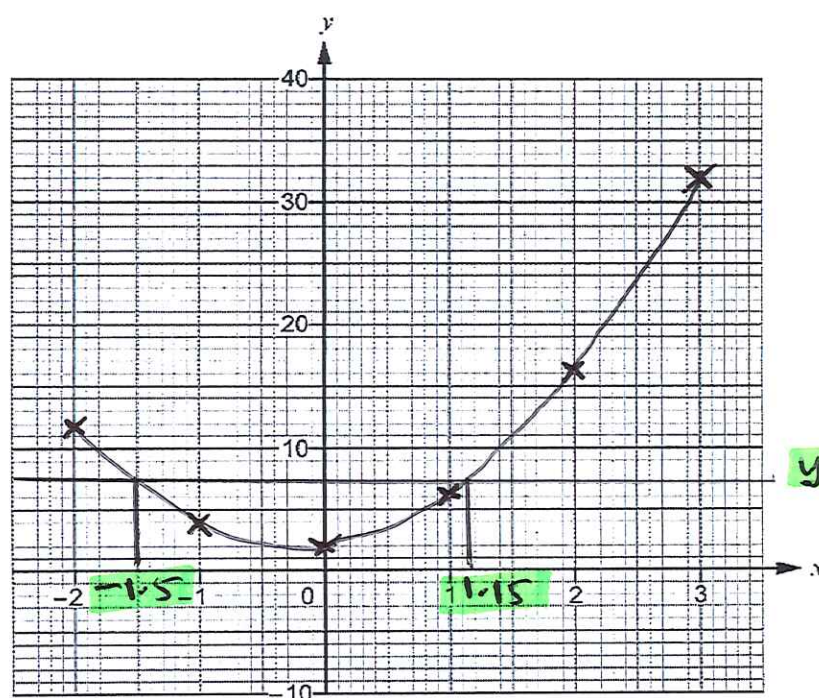
x	-2	-1	0	1	2	3
$y = 3x^2 + x + 2$	12	4	2	6	16	32

$$x = -1, y = 3(-1)^2 + (-1) + 2 = 4$$

$$x = 2, y = 3(2)^2 + 2 + 2 = 16$$

(b) On the graph paper below, draw the graph of $y = 3x^2 + x + 2$ for values of x from -2 to 3 .

[2]



(c) Use your graph to solve the equation $3x^2 + x + 2 = 7$.

[2]

We have drawn $y = 3x^2 + x + 2$

If we subtract $y = 7$

$$0 = 3x^2 + x + 2 - 7$$

$$7 = 3x^2 + x + 2$$

$$x = 1.15 \text{ and } x = -1.5$$

The examiner will follow through from your graph and will accept

$$-1.4 \text{ to } -1.5 \text{ and } 1.1 \text{ to } 1.2$$

4.

The following table shows values of $y = x^2 - 2x - 3$ for values of x from -2 to 5 .

x	-2	-1	0	1	2	3	4	5
y	5	<u>0</u>	-3	-4	-3	0	5	<u>12</u>

(a) Complete the table by finding the values of y when $x = -1$ and $x = 5$.

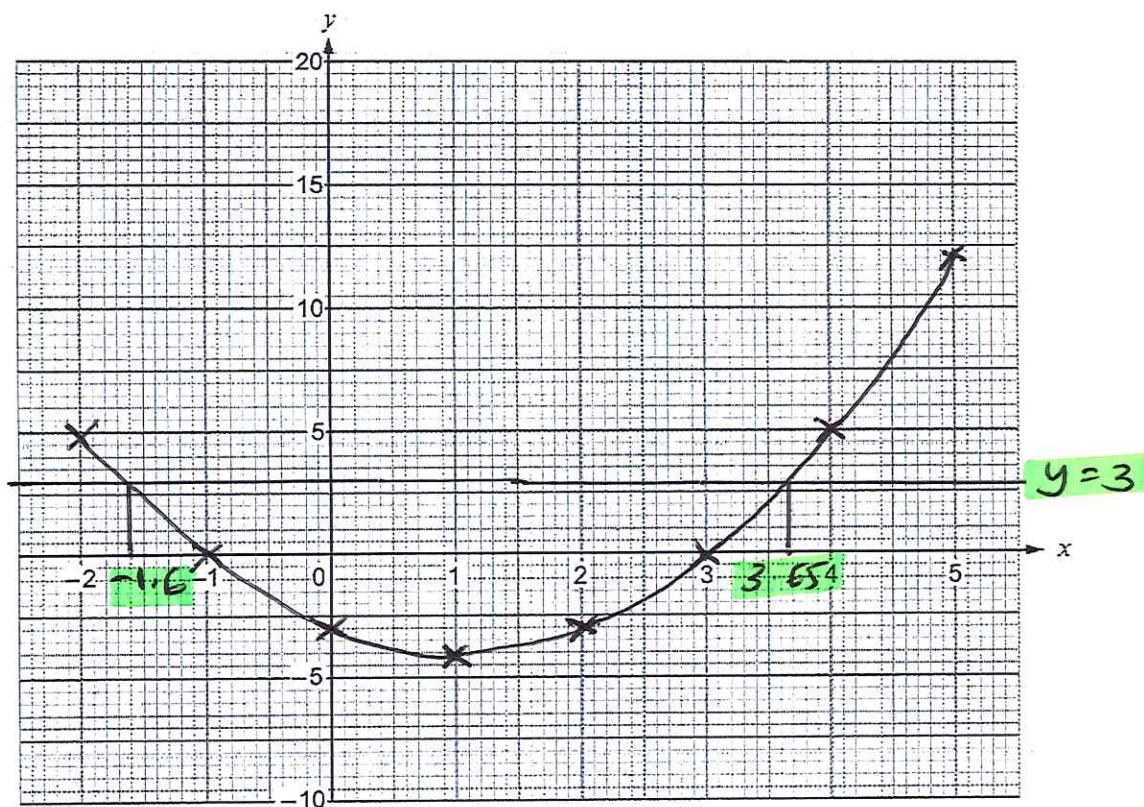
[2]

$$\text{If } x = -1, y = (-1)^2 - 2(-1) - 3, y = 0$$

$$\text{If } x = 5, y = (5)^2 - 2(5) - 3, y = 12$$

(b) Use the graph paper below to sketch the graph of $y = x^2 - 2x - 3$.

[2]



(c) Draw the line $y = 3$ on the same graph paper.

[1]

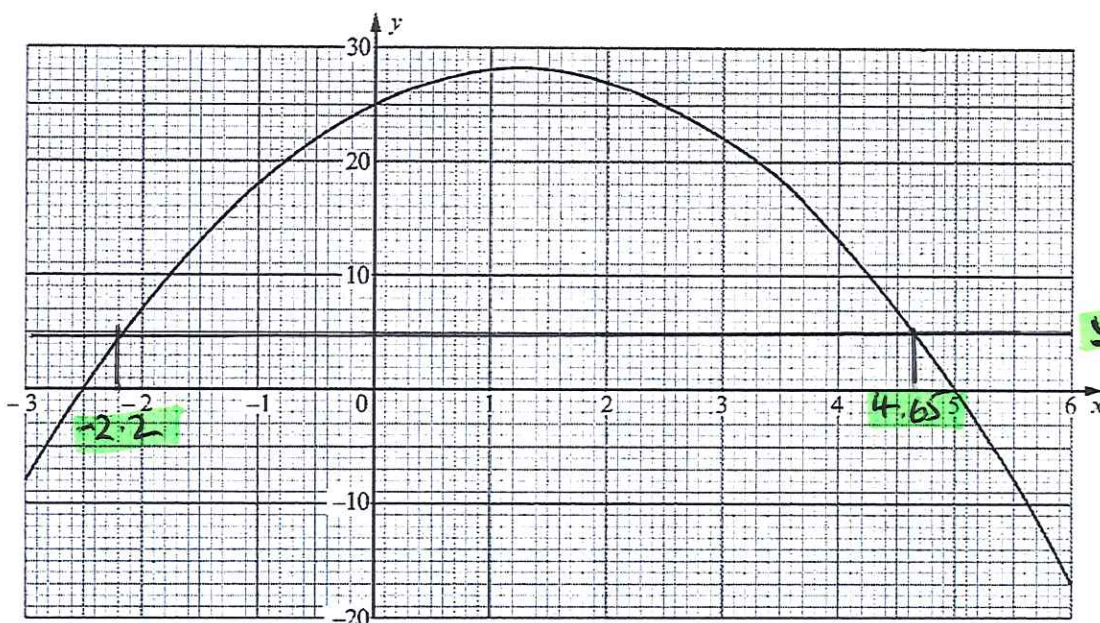
(d) Write down the x -coordinates of the points where the curve $y = x^2 - 2x - 3$ intersects the line $y = 3$.

[1]

$$x = -1.6 \text{ and } x = 3.65$$

5.

The graph of $y = -2x^2 + 5x + 25$ for values of x from -3 to 6 is shown below.



(a) Use the graph to solve each of the following equations.

(i) $-2x^2 + 5x + 25 = 0$

$x = -2.2$ and $x = 5$

[1]

(ii) $-2x^2 + 5x + 20 = 0$

$-2x^2 + 5x + 25 = 5$

draw on $y = 5$

$x = 4.65$ and $x = -2.2$

[2]

examiner accepts -2.2 to -2.1 and 4.6 to 4.7

(b) Find the coordinates of the points of intersection of the graph of $y = -2x^2 + 5x + 25$ and the graph of $y = x^2 - 2x - 3$.

equating gives $x^2 - 2x - 3 = -2x^2 + 5x + 25$

$3x^2 - 7x - 28 = 0$

$\times ac$	$+b$
-84	-7

$a=3, b=-7, c=-28$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

won't factorise

$x = \frac{7 \pm \sqrt{49 - 4(3)(-28)}}{2(3)}, x = \frac{7 \pm \sqrt{385}}{6}$

$x = 4.4$ and $x = -2.1$

substitute back into original equation to find y .

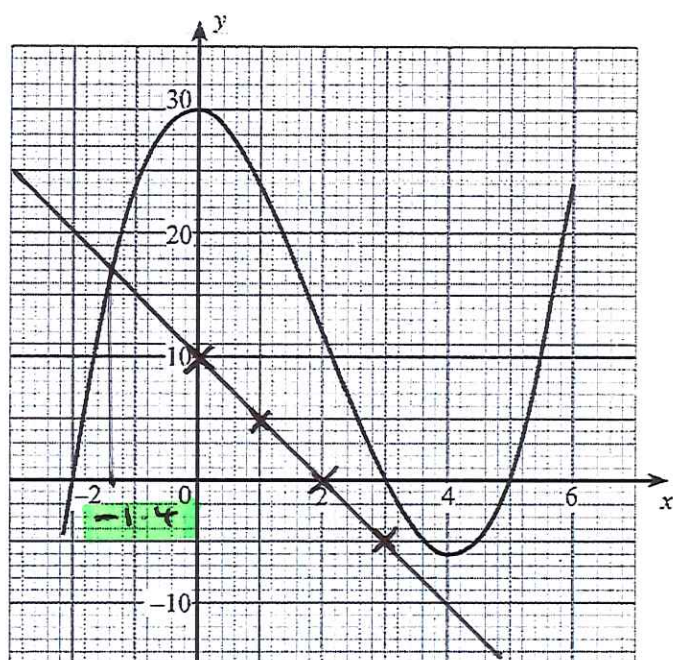
$(4.4, 7.8)$ and $(-2.1, 5.6)$

[5]

OR you could have plotted $y = x^2 - 2x - 3$

6.

The graph of the equation $y = x^3 - 6x^2 - x + 30$ is shown on the graph paper below.



$$y = -5x + 10$$

Use the graph above to answer the following questions.

(a) Solve $x^3 - 6x^2 - x + 30 = 0$.

$$\underline{x = -2}, \underline{x = 3}, \underline{x = 5}$$

[2]

(b) By drawing a suitable straight line, solve the equation $x^3 - 6x^2 - x + 30 = -5x + 10$.

Require $y = -5x + 10$

x	0	1	2	3
y	10	5	0	-5

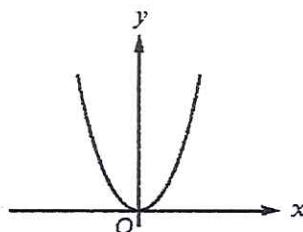
$$\underline{x = -1.4}$$

examiner accepts
-1.5 to -1.3

[4]

7.

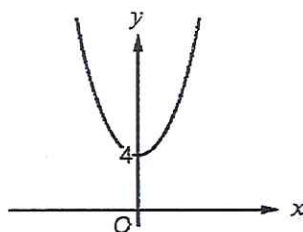
Madge sketches the following graph.



She correctly says:

"My graph is a sketch of $y = x^2$."

Leon sketches the following graph.



He also makes a correct statement.

- (a) Complete Leon's statement with a correct equation.

"My graph is a sketch of $y = \underline{x^2 + 4}$."

[1]

or $\underline{kx^2 + 4}$
where
 $\underline{k > 0}$

- (b) Madge and Leon notice their graphs have no negative values of y .

Daisy also tries to sketch a graph.

Her graph is a sketch of $y = x^2 - 25$.

Should Daisy's graph have any negative values of y ?

Tick a box.

Yes ☒ No ☐ Don't know ☐

Give a reason for your answer.

[1]

Yes because it will be a translation
25 down/in the negative y direction so
some of Daisy's graph will be below the
 y axis.

or

yes because for example if $x = 0$
when $y = x^2 - 25$
 $y = -25$ $(0, -25)$
↓
below y axis

8.

The table shows values of $y = 3x^2 + 2x - 10$ for values of x from -4 to 3 .

x	-4	-3	-2	-1	0	1	2	3
$y = 3x^2 + 2x - 10$	30	11	-2	-9	-10	-5	6	23

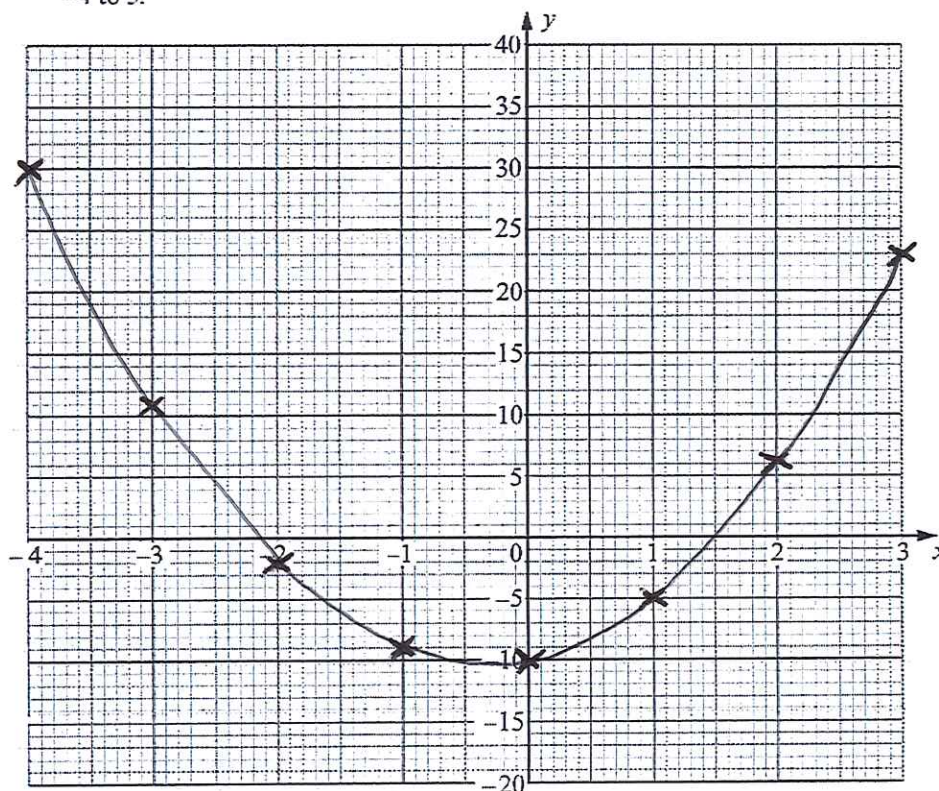
(a) Complete the table above.

when $x = -3$ $y = 3(-3)^2 + 2(-3) - 10$
 $y = 11$

[1]

(b) On the graph paper below, draw the graph of $y = 3x^2 + 2x - 10$ for values of x from -4 to 3 .

[2]

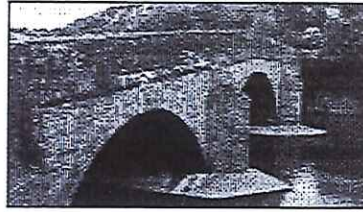


(c) Write down the x -coordinates of the points where the graph of $y = 3x^2 + 2x - 10$ intersects the x -axis.

$x = -2.5$ and $x = 1.5$

[1]

examine will follow through from your graph.



When a stone is dropped from a bridge into the river below, the equation for calculating the height of the bridge is given as

$$H = \frac{1}{2} g t^2.$$

In this equation:

- H is the height of the bridge in metres
- g is the acceleration of the stone due to gravity, which is measured in m/s^2
- $g = 9.8$
- t is the time taken in seconds for the stone to hit the surface of the water.

Glynis carries out experiments on different bridges.

The highest bridge she uses in her experiment is 200m above the river level.

you only need to
go up to $H = 200\text{m}$.

Because of my
scale I went

upto

$t = 8$

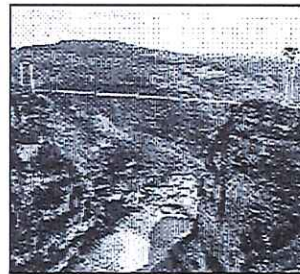
but

could

have

stopped

at $t = 7$

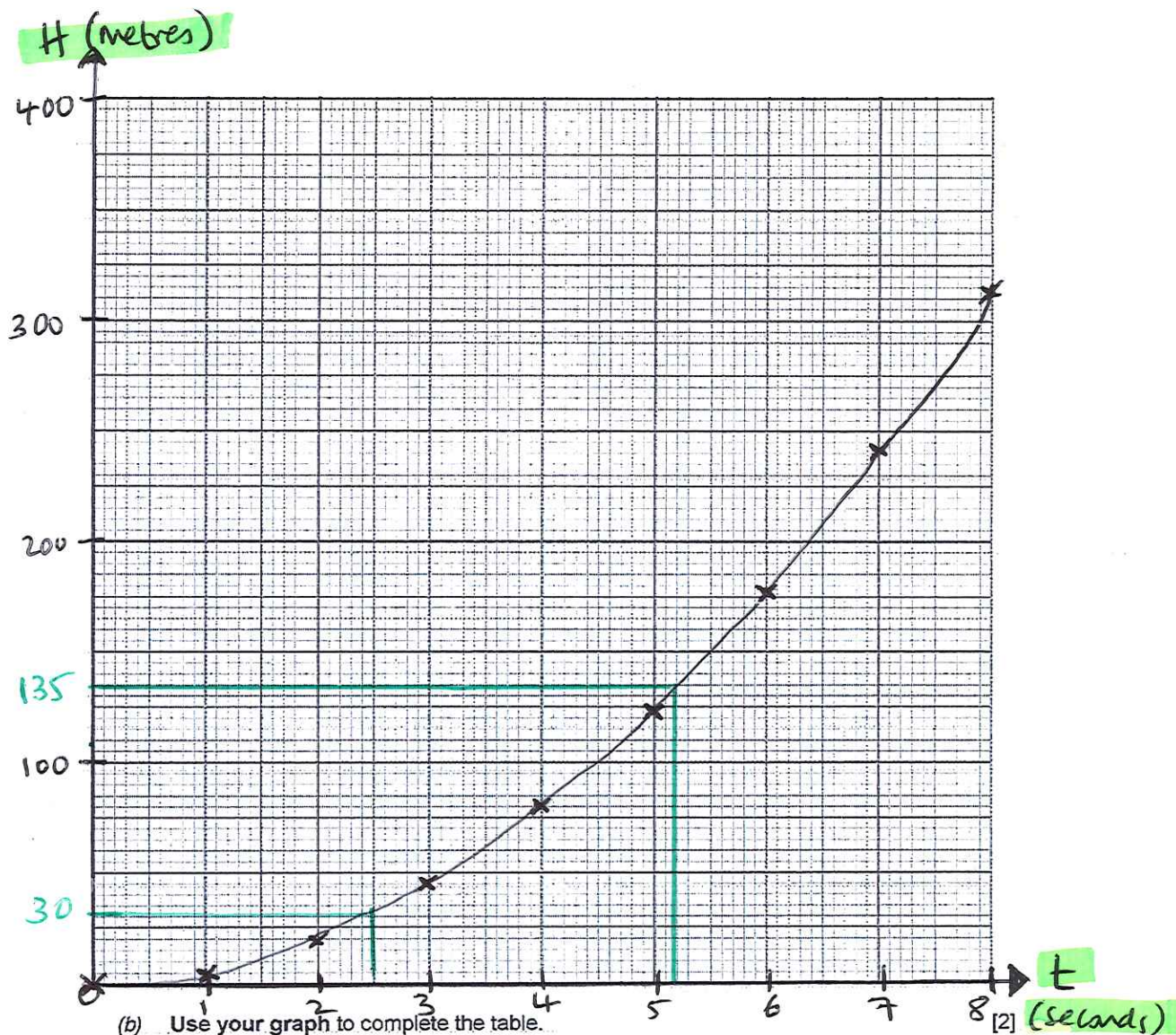


Glynis drops a stone from each bridge and times how long it takes to hit the water. These times allow Glynis to find the heights of the bridges.

She decides that a graph would be a helpful way of finding the heights of the bridges.

- (a) Use the graph paper opposite to draw a graph for Glynis to be able to read off the heights of various bridges. [5]

t	0	1	2	3	4	5	6	7	8
H	0	4.9	19.6	44.1	78.4	122.5	176.4	240.1	313.6



Time taken for stone to drop, in seconds	Height of bridge, in metres	Height of bridge, in kilometres written in standard form
2.5	<u>30</u>	<u>3×10^{-2}</u>
5.2	<u>135</u>	<u>1.35×10^{-1}</u>

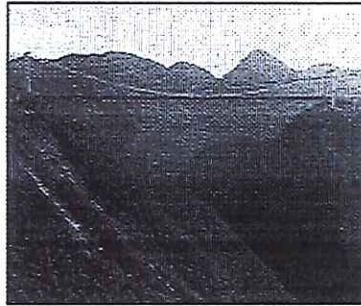
$$30 \text{ metres} = 30 \div 1000 \text{ km} = 0.03 = 3 \times 10^{-2} \text{ km}$$

$$135 \text{ metres} = 135 \div 1000 \text{ km} = 0.135 = 1.35 \times 10^{-1} \text{ km}$$

- (c) Glynis thinks there is a risk that there are some errors in her data.
State a possible error.

[1]

- The time taken might not have been accurately measured
 - Error could be due to changes in river level
 - Windy
 - Air resistance neglected (any of these)
 - Different stones
- (d) In 2009 the Sidu River Bridge in China became the world's highest river bridge.
Its height is 496 metres.



Calculate, using the formula $H = \frac{1}{2}gt^2$, how long it would take a stone dropped from the bridge to hit the surface of the Sidu River below.
You must show all your working.

[3]

$$496 = \frac{1}{2} \times 9.8 \times t^2$$

$$\frac{496 \times 2}{9.8} = t^2$$

$$t = \sqrt{101.224 \dots} = 10.061 \dots$$

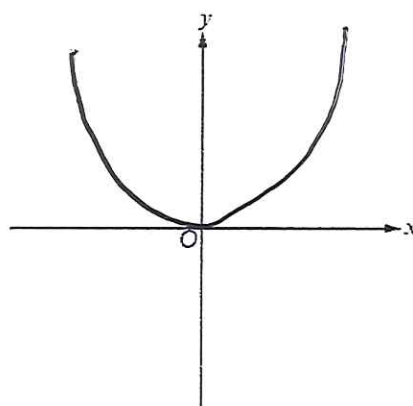
$$\underline{t = 10.1 \text{ seconds (1dp)}}$$

10.

Use the axes given below to sketch the following.

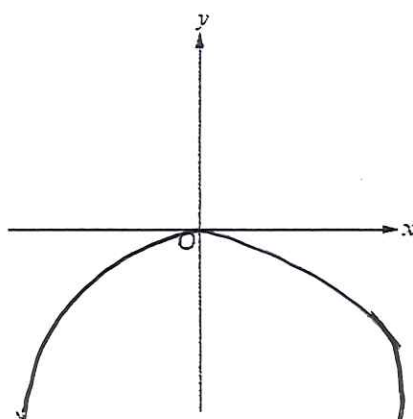
(a) $y = x^2$

[1]



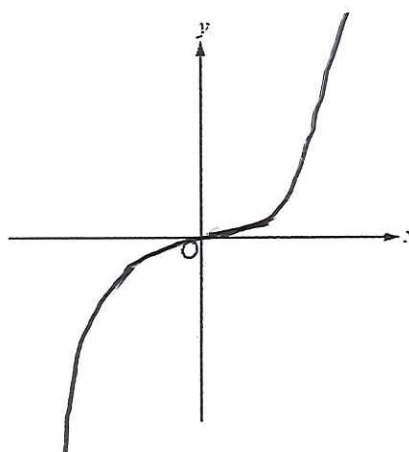
(b) $y = -x^2$

[1]



(c) $y = x^3$

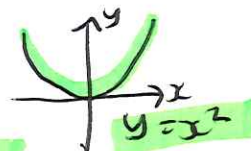
[1]



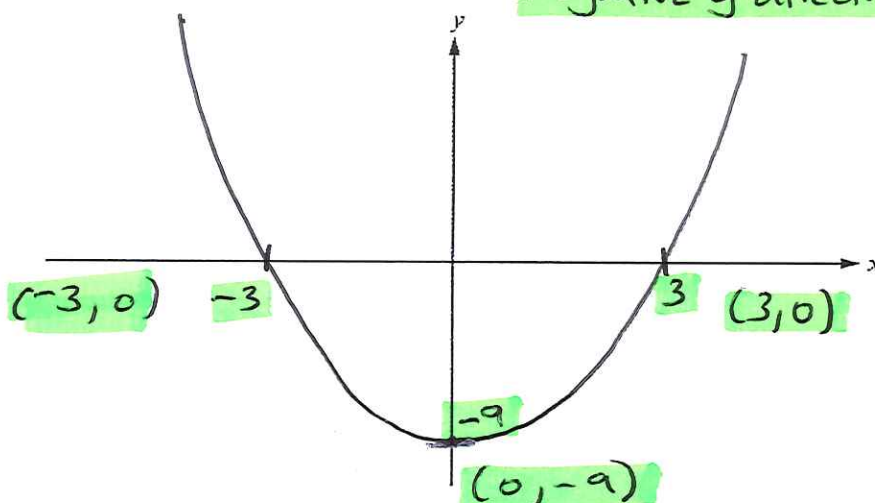
You'll notice my curve sketches aren't great. I'd get away with them (just!). Be as careful as you can.

11.

Think of $y = x^2$ first
then $y = x^2 - 9$ is a



- (a) Use the axes below to sketch $y = x^2 - 9$. translation 9 down
Mark clearly the coordinates of any point where this curve meets an axis. (negative y direction) [3]



$$y = x^2 - 9$$

$$y = (x - 3)(x + 3)$$

$$x = 3 \text{ and } x = -3 \text{ when } y = 0$$

- (b) Mari is asked to sketch $y = (x + 2)^2 - 9$.
Describe how Mari could use your sketch from (a) to sketch this curve. [2]

Translation, 2 to the left, x direction

or

shift -2 horizontally

or move to the left 2

12.

Gerry has sketched the floor plan of a room.

All the corners of the room are either 90° or 270° .

Gerry has forgotten some of the measurements but she knows two of the measurements are the same, so she has labelled them x metres.

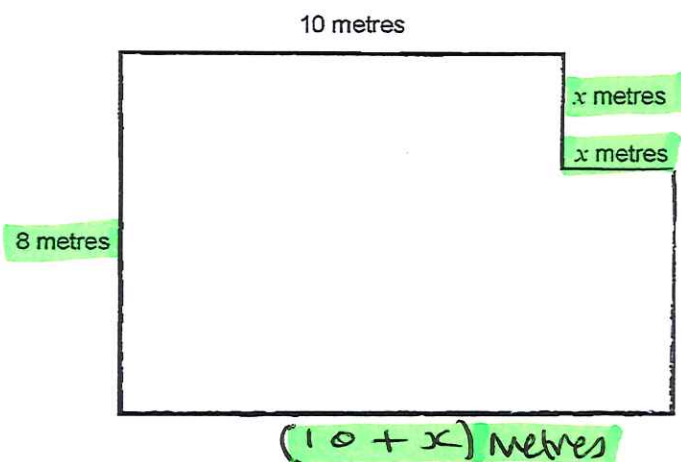


Diagram not drawn to scale

- (a) Show that the area of the floor, in square metres, is $80 + 8x - x^2$. [2]

Area = Big rectangle - Small rectangle

$$= (8 \times (10 + x)) - (x \times x)$$

$$= \underline{80 + 8x - x^2}$$

- (b) Use the graph paper opposite to draw the graph representing the area, in square metres, of the floor plan.
Use values of x from $x = 0$ to $x = 5$. [3]

The area is $80 + 8x - x^2$ square metres.

Let $y = 80 + 8x - x^2$

Require

x	0	1	2	3	4	5
y	80	87	92	95	96	95

$x = 0$, $y = 80$

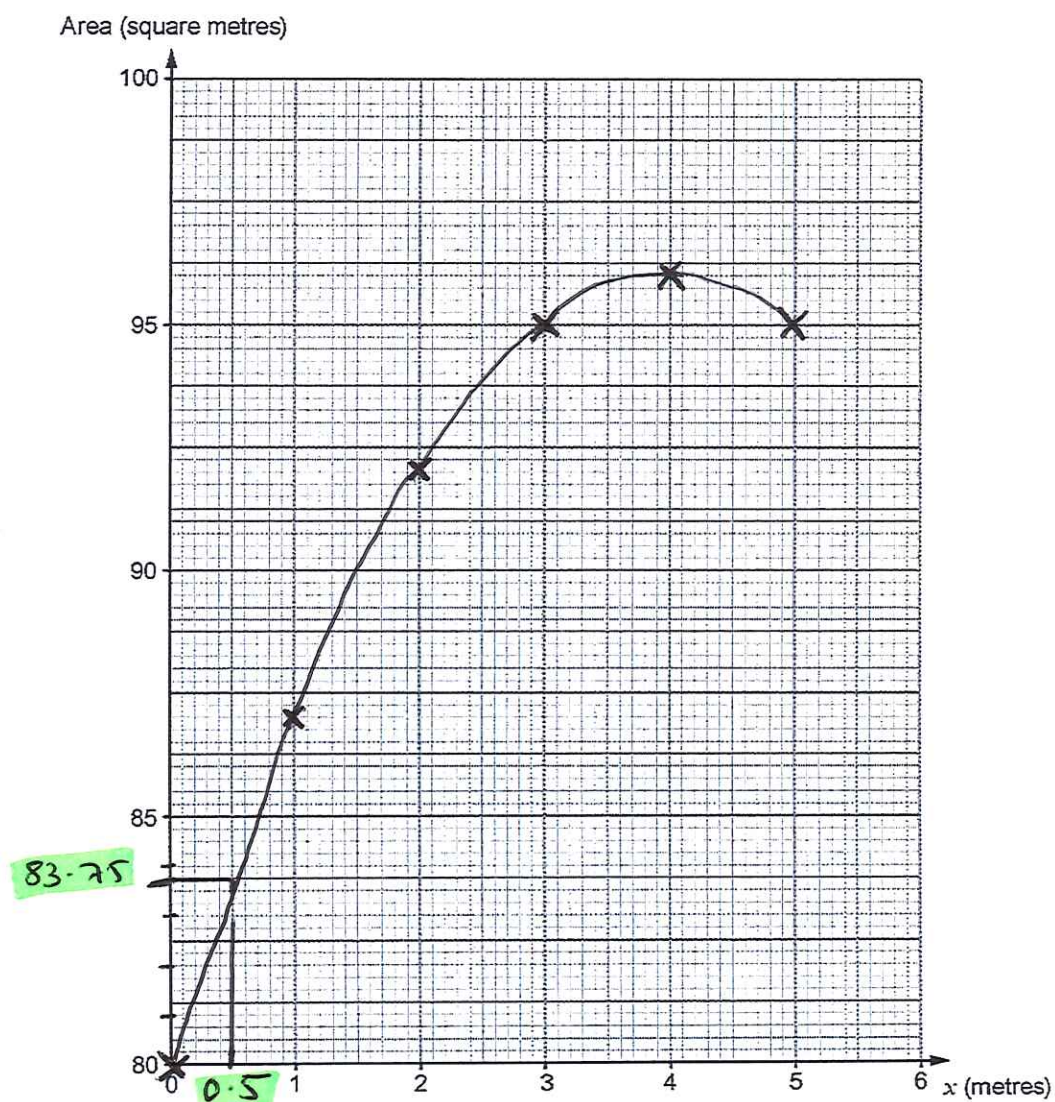
$x = 1$, $y = 80 + 8(1) - (1)^2 = \underline{87}$

$x = 2$, $y = 80 + 8(2) - (2)^2 = \underline{92}$

$x = 3$, $y = 80 + 8(3) - (3)^2 = \underline{95}$

$x = 4$, $y = 80 + 8(4) - (4)^2 = \underline{96}$

$x = 5$, $y = 80 + 8(5) - (5)^2 = \underline{95}$



- (c) Gerry calculated the area of the floor in the room before she lost some of measurements.
The area is 83.75 square metres. Find the value of x .

[2]

When $x = 0.5$ the area = 83.75