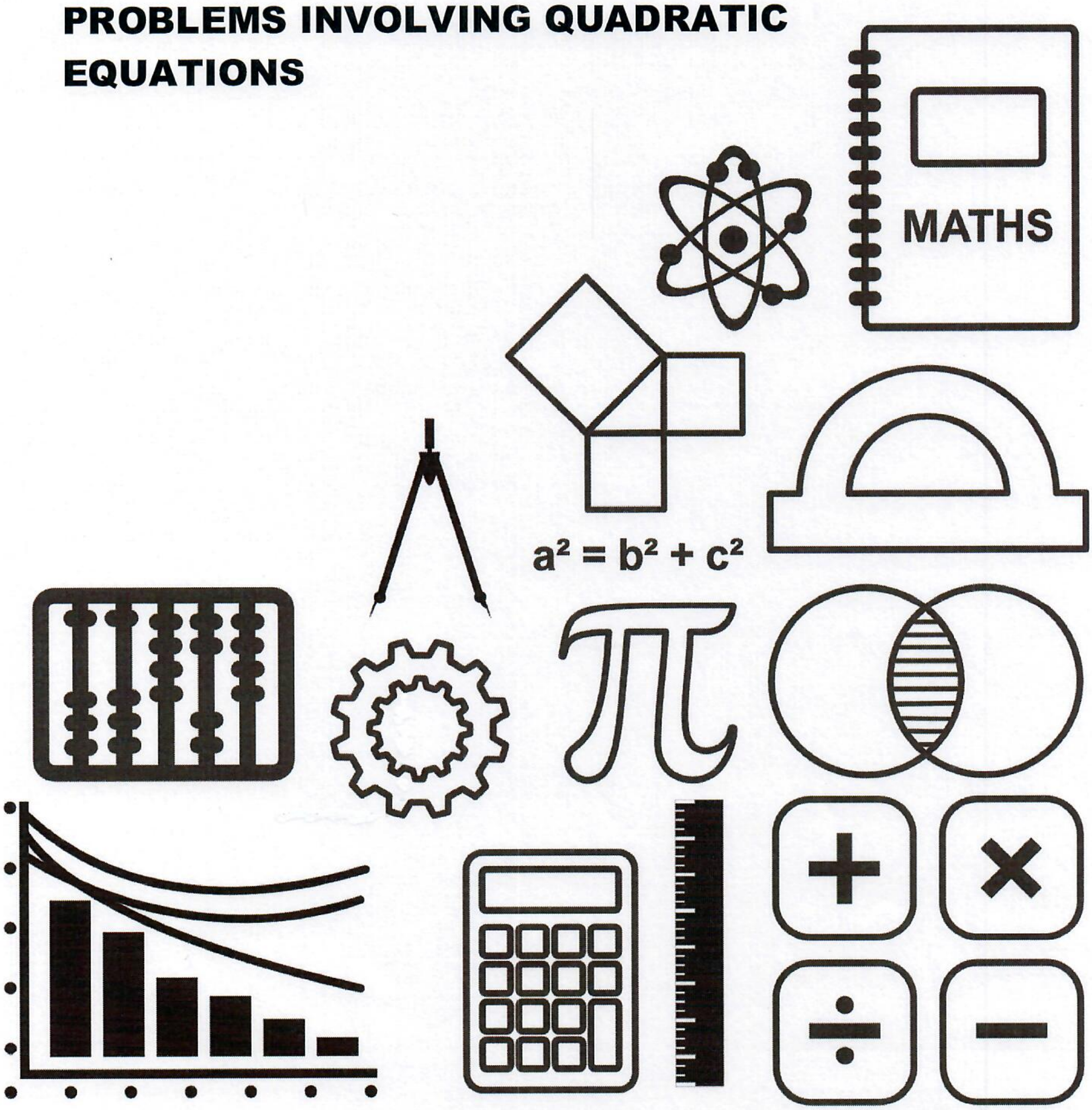
MATHSDIY

SOLUTIONS

GCSE TOPIC BOOKLET
PROBLEMS INVOLVING QUADRATIC EQUATIONS



1.

The base of an open rectangular box is of length $(2x + 6)$ cm and width x cm.
The area of this base is $59\,\text{cm}^2$.
The height of the open box is $(x - 3)$ cm.

(a) Show that $2x^2 + 6x - 59 = 0$. [2]

Area of base = length × width

$(2x+6)(x) = 59$

$2x^2 + 6x - 59 = 0$

→ won't factorise use formula

(b) (i) Solve the equation $2x^2 + 6x - 59 = 0$, giving your answers correct to 2 decimal places.
You must show all your working. [3]

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $a = 2$, $b = 6$, $c = -59$

$x = \dfrac{-6 \pm \sqrt{6^2 - 4(2)(-59)}}{2(2)}$

$x = \dfrac{-6 \pm \sqrt{508}}{4}$

$x = 4.13$ (2dp) $x = -7.13$ (2dp)

(ii) Hence calculate the volume of the box.
State clearly the units of your answer. [3]

width $= x = 4.13$ (cannot have length $= -7.13$)

length $2x+6 = 2 \times 4.13 + 6 = 8.26 + 6$
$= 14.26$

height $= x - 3 = 4.13 - 3 = 1.13$

$V = L \times W \times H = 14.26 \times 4.13 \times 1.13$
$= 66.5\,\text{cm}^3$

Volume of the box is $66.5\,\text{cm}^3$

examiner accepts answers in the range 65.5 - 67 (because of rounding)

2. A rectangle of length $(x + 8)$ cm and width x cm has an area of y cm^2.
It is known that $y - x = 1284$.
Find the dimensions of the rectangle.
Give your answer correct to 1 decimal place. → use quadratic formula as won't factorise
You must use an algebraic method.

Area $= L \times W = (x+8)(x) = y$

$$x^2 + 8x = y \quad (1)$$

$$y - x = 1284$$

$$y = 1284 + x \quad (2)$$

Substitute (2) into (1) to get

$$x^2 + 8x = 1284 + x$$

$$x^2 + 7x - 1284 = 0$$

$a = 1$
$b = 7$
$c = -1284$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-7 \pm \sqrt{(7)^2 - 4(1)(-1284)}}{2(1)}$$

[8]

$$x = \frac{-7 \pm \sqrt{5185}}{2}$$

$x = 32.5034\ldots$ or $x = -39.5034\ldots$

$x = 32.5$ cm

$x = -39.5$
discard as can't have negative length

Require dimensions
length $= x + 8 = 32.5 + 8 = 40.5$ cm
width $= x = 32.5$ cm

3.

The diagram shows a parallelogram and a rectangle joined along a common side.

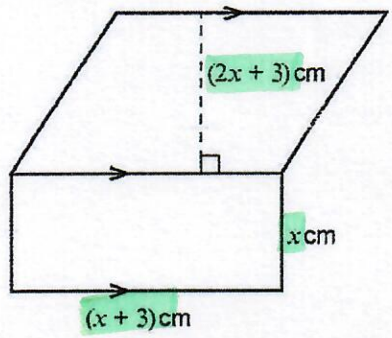


Diagram not drawn to scale

The width of the rectangle is x cm.
The length of the rectangle is $(x + 3)$ cm.
The height of the parallelogram is $(2x + 3)$ cm.
The total area of the parallelogram and the rectangle together is 70 cm^2.

(a) Show that $3x^2 + 12x - 61 = 0$. [3]

area rectangle $= L \times W = x(x+3)$

area parallelogram $= b \times h = (x+3)(2x+3)$

Total area $= (x+3)(2x+3) + x(x+3) = 70$

$2x^2 + 6x + 3x + 9 + x^2 + 3x = 70$

$3x^2 + 12x + 9 - 70 = 0$

$\underline{3x^2 + 12x - 61 = 0}$

$a = 3$
$b = 12$
$c = -61$

(b) Use the quadratic formula to calculate the length of the rectangle. Give your answer correct to 2 decimal places. [4]

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \dfrac{-12 \pm \sqrt{12^2 - 4(3)(-61)}}{2(3)}$

$x = \dfrac{-12 \pm \sqrt{876}}{6}$

$x = 2.9328\ldots$ $\quad x = -6.9328\ldots$

discard as cannot have a negative length.

$\underline{x = 2.93 \text{ cm } (2dp)}$

length of rectangle $= x + 3$
$= 2.93 + 3$
$= \underline{5.93 \text{ cm}}$

4. A right-angled triangle is shown below.

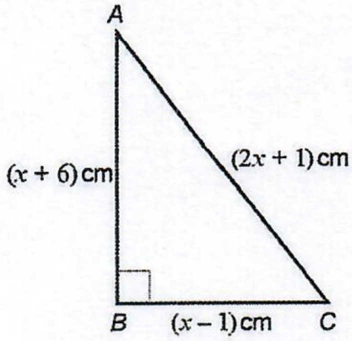


Diagram not drawn to scale

(a) Show that $x^2 - 3x - 18 = 0$. [4]

Using pythagaras $B^2 = A^2 + C^2$

$(2x+1)^2 = (x-1)^2 + (x+6)^2$

$(2x+1)(2x+1) = (x-1)(x-1) + (x+6)(x+6)$

$4x^2 + 4x + 1 = x^2 - 2x + 1 + x^2 + 12x + 36$

$2x^2 - 6x - 36 = 0 \quad (\div 2)$

$x^2 - 3x - 18 = 0$

(b) Factorise the expression $x^2 - 3x - 18$, and hence solve the equation $x^2 - 3x - 18 = 0$. Write down the lengths of the sides of the right-angled triangle. [4]

$x^2 - 3x - 18 = (x-6)(x+3)$

$x^2 - 3x - 18 = 0$

$(x-6)(x+3) = 0$

$x - 6 = 0$ and $x + 3 = 0$

$x = 6$ $\quad$ $x = -3$ ← discard

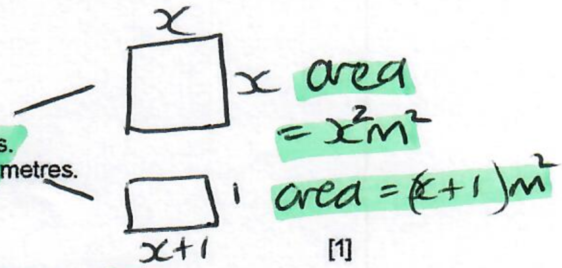
AB = 12 cm AC = 13 cm BC = 5 cm

$AB = x + 6 = 12$

$AC = 2x + 1 = 2(6) + 1 = 13$

$BC = x - 1 = 6 - 1 = 5$

5. Aled has three concrete slabs.
Two of the slabs are square, with each side of length x metres.
The third slab is rectangular and measures 1 metre by $(x+1)$ metres.
The three concrete slabs cover an area of $7\,\text{m}^2$.



(a) Show that $2x^2 + x - 6 = 0$. [1]

Area of 2 squares + Area rectangle = 7

$2 \times (x^2) + (x+1) = 7$

$2x^2 + x + 1 - 7 = 0$ $\qquad \underline{\underline{2x^2 + x - 6 = 0}}$

(b) Solve the equation to find the length of each side of the square slabs.
You must justify any decisions that you make. [4]

$\times ac$	$+b$
-12	1

$-3, 4$

$2x^2 + x - 6 = 0$

$2x^2 + 4x - 3x - 6 = 0$

$2x(x+2) - 3(x+2) = 0$

$(x+2)(2x-3) = 0$

$\underline{\underline{x = -2}}$ and $\underline{\underline{x = 3/2}}$

discard $x = -2$ as cannot have a negative length.

$\underline{\underline{x = 1.5}}$

The length of the side of the square slab $\underline{\underline{= 1.5 \text{ metres}}}$

6. Wilf thinks of a number, x.

His sister says that if Wilf subtracts 6 from his number and multiplies this new number by the number he first thought of, he will get an answer of –5.

Use this information to

- find a quadratic equation in the form $x^2 + ax + b = 0$, and
- hence solve the equation to find the possible values of x. [6]

$(x-6)(x) = -5$

$x^2 - 6x + 5 = 0$

$(x-5)(x-1) = 0$

$x = 1$ and $x = 5$

7.

The triangle below has an area of $10\,\text{cm}^2$.
Calculate the value of x. [7]

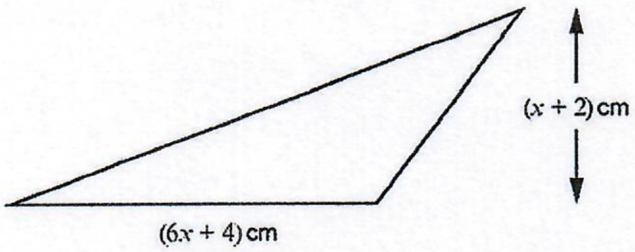


Diagram not drawn to scale

$\text{Area } \Delta = \frac{1}{2}(b \times h)$

$10 = \frac{1}{2}(6x+4)(x+2)$

$20 = (6x+4)(x+2)$

$20 = 6x^2 + 4x + 12x + 8$

$0 = 6x^2 + 16x - 12 \quad (\div 2)$

$3x^2 + 8x - 6 = 0$

Try and factorise

$\times$	$+$
ac	b
-18	8

won't work

Must ∴ use formula

$a = 3$
$b = 8$
$c = -6$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-8 \pm \sqrt{8^2 - 4(3)(-6)}}{2(3)}$$

$$x = \frac{-8 \pm \sqrt{136}}{6}$$

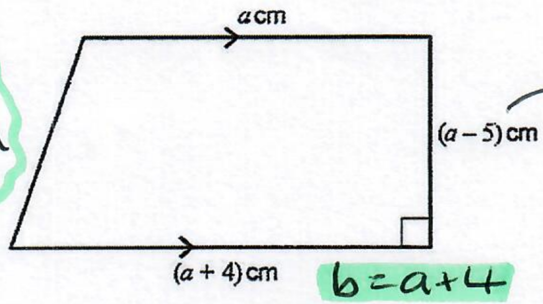
$x = 0.6103\ldots$
$x = -3.276\ldots$ ← discard
can't have negative length

$\underline{\underline{x = 0.61}}$ (2dp)

8. The diagram shows a trapezium.

area of a trapezium $= \frac{1}{2}(a+b)\times h$

$a=a$



$h=a-5$

$b=a+4$

Diagram not drawn to scale

(a) Show that the area of the trapezium is $(a^2 - 3a - 10)$ cm^2.
You must show all your working. [4]

$$\text{area} = \frac{1}{2}(a+a+4)(a-5)$$

$$= \frac{1}{2}(2a+4)(a-5)$$

$$= (a+2)(a-5)$$

$$= a^2+2a-5a-10$$

$$= \underline{\underline{a^2-3a-10}}$$

(b) The area of the trapezium is 30 cm^2.
Use an algebraic method to calculate the height of the trapezium. [5]

$$a^2-3a-10=30$$

$$a^2-3a-40=0$$

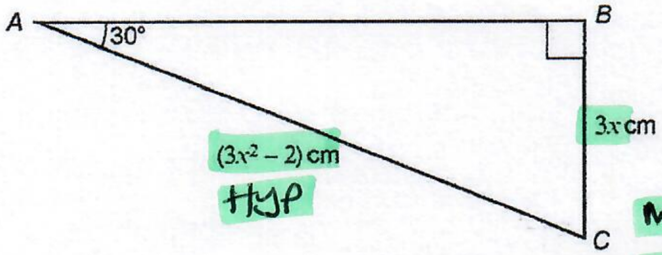
$$(a-8)(a+5)=0$$

$a=8$ and $a=-5$ discard as can't have a negative length

$\underline{\underline{a=8}}$

$\underline{\underline{\text{height}}} = a-5 = 8-5 = \underline{\underline{3\text{cm}}}$

9. (a) In the triangle below, show that x satisfies the equation $3x^2 - 6x - 2 = 0$. [3]



Must be right angled trigonometry

Diagram not drawn to scale

$\sin 30 = \dfrac{\text{opp}}{\text{hyp}}$

$\sin 30 = \dfrac{3x}{(3x^2-2)}$ but $\sin 30 = \dfrac{1}{2}$

$\therefore \dfrac{1}{2} = \dfrac{3x}{(3x^2-2)}$

$3x^2 - 2 = 2(3x)$

$3x^2 - 6x - 2 = 0$

(b) Solve the equation $3x^2 - 6x - 2 = 0$ and hence find the length of *BC*, correct to 1 decimal place. [4]

won't factorise, use formula

$a = 3 \quad b = -6 \quad c = -2$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $\qquad x = \dfrac{6 \pm \sqrt{(-6)^2 - 4(3)(-2)}}{2(3)}$

$x = \dfrac{6 \pm \sqrt{60}}{6}$

$x = 2.29$ or $x = -0.29$

discard as can't have negative length

$BC = 3x$

$BC = 3 \times (2.29)$

$BC = 6.87$

$BC = 6.9\text{cm}$ (1dp)