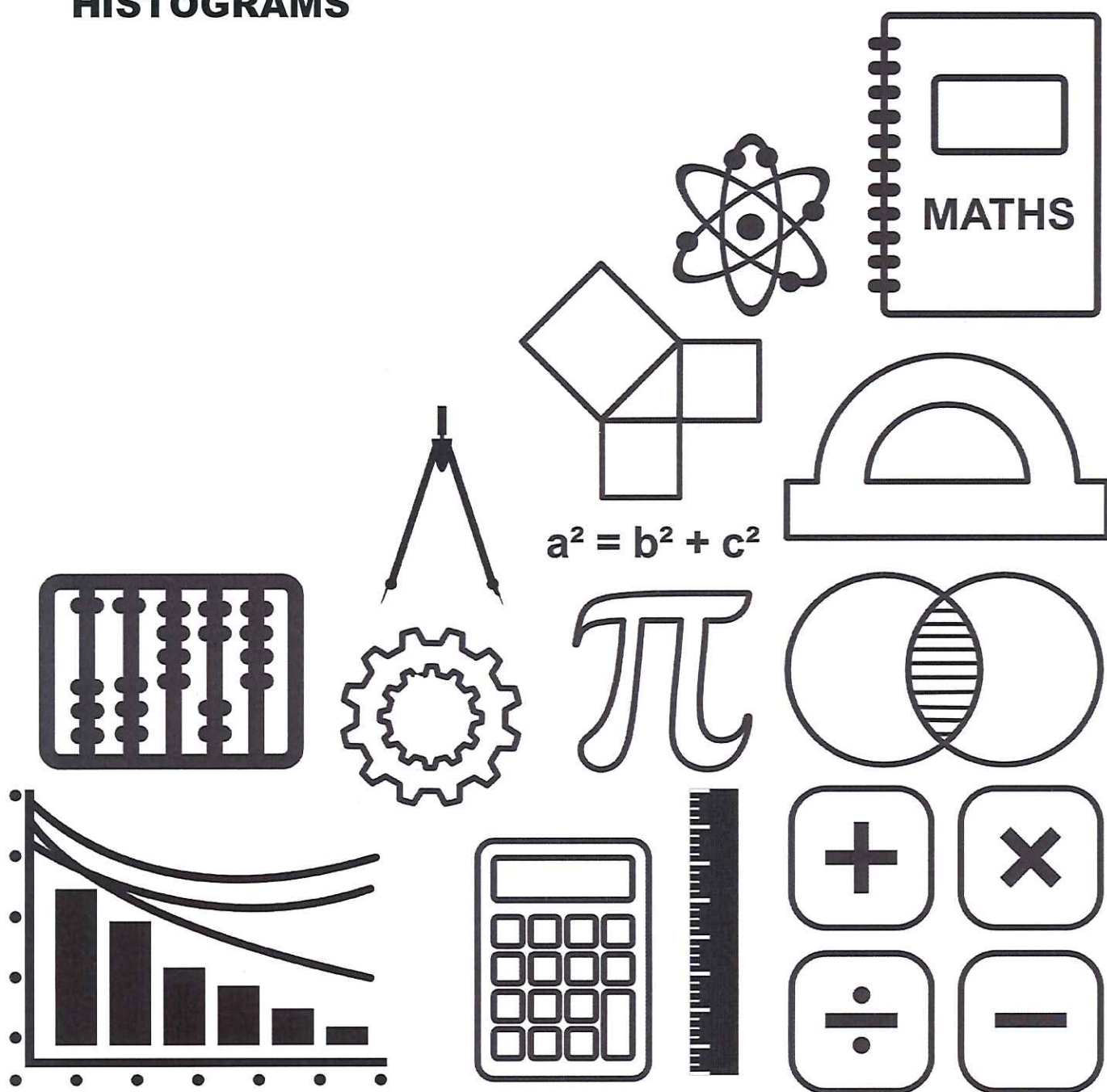


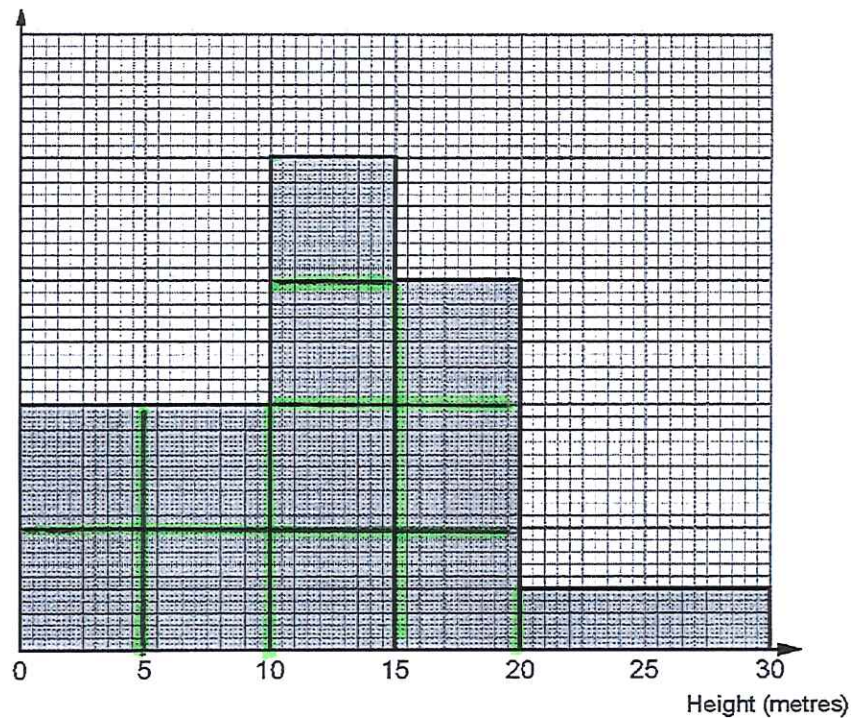
GCSE TOPIC BOOKLET HISTOGRAMS



1.

This histogram shows the distribution of heights, in metres, of 240 trees in a wood.

Frequency density



How many trees were between 10 and 15 metres in height?

[3]

Strategy:- realise that there are 12 'blocks'

and the area of all 12 = 240 trees

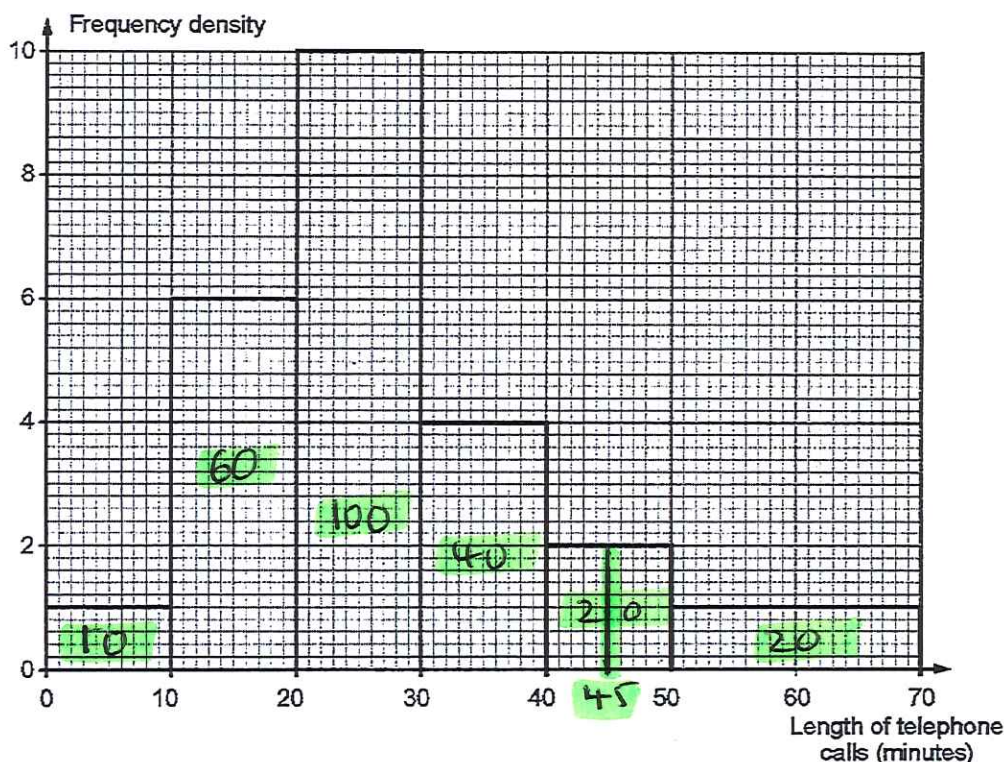
$\therefore$ area of 1 block = $240 \div 12 = 20$ trees

4 blocks between 10 and 15 $\therefore 4 \times 20$

= 80 trees

2.

The histogram illustrates the lengths of telephone calls made to a computer helpline during one Friday evening.



- (a) Calculate how many telephone calls were made to the computer helpline during the Friday evening. [3]

$$\begin{aligned}
 & (10 \times 1) + (10 \times 6) + (10 \times 10) + (10 \times 4) + (10 \times 2) + (20 \times 1) \\
 &= 10 + 60 + 100 + 40 + 20 + 20 \\
 &= \underline{\underline{250 \text{ phone calls}}}
 \end{aligned}$$

- (b) Calculate an estimate of the percentage of telephone calls that lasted longer than $\frac{3}{4}$ of an hour. [2]

$$\begin{aligned}
 & \frac{3}{4} \text{ of an hour} = \underline{\underline{45 \text{ minutes}}} \\
 &= \left(\frac{1}{2} \text{ of the phone calls between } 40-50 \right) + \left(\text{all the } 60-70 \text{ calls} \right) \\
 &= 10 + 20 = \underline{\underline{30 \text{ phone calls over } 45 \text{ minutes}}} \\
 & \% = \frac{30}{250} \times 100 = \underline{\underline{12\%}}
 \end{aligned}$$

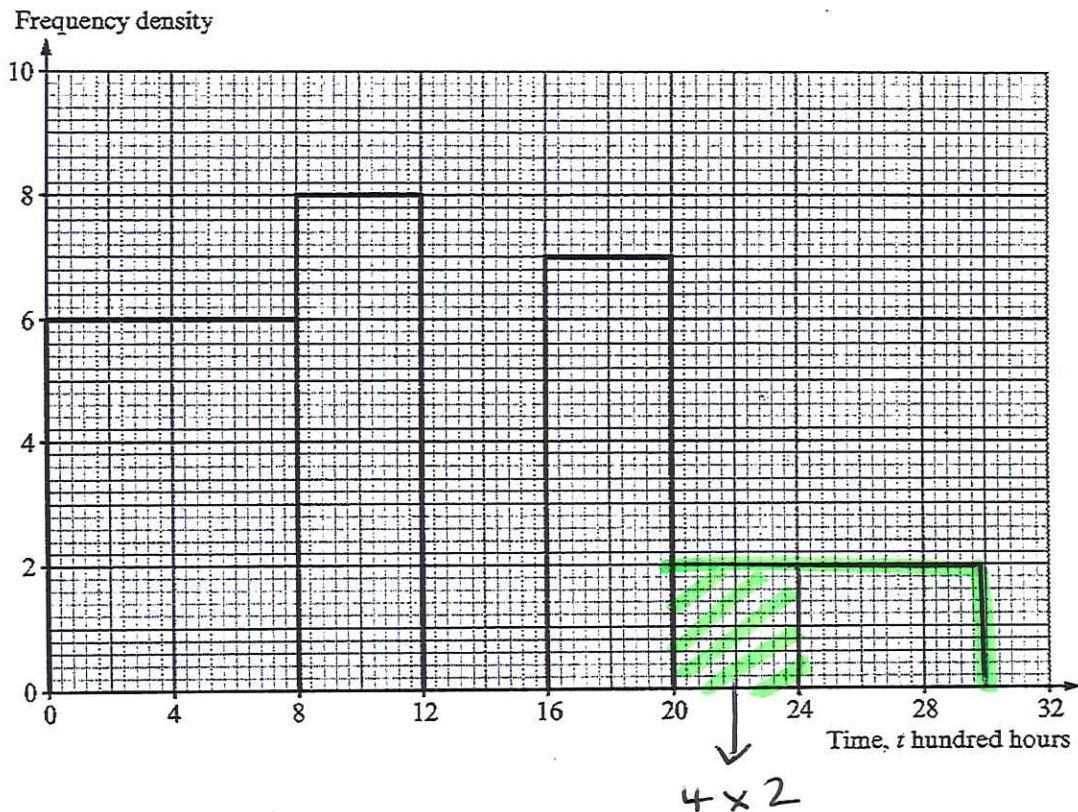
- (c) Estimate the median length of a telephone call made to the computer helpline during the Friday evening. Give your answer correct to 1 decimal place. [3]

$$\begin{aligned}
 & \underline{\underline{\text{Median} = \text{middle} = 125^{\text{th}} \text{ call}}} \\
 & \text{By inspection } 70 \text{ calls below } 20 \text{ minutes } \therefore 125^{\text{th}} \\
 & \text{call} = \left(\frac{55}{100} \times 10 \right) + 20 = \underline{\underline{25.5 \text{ minutes}}} \\
 & \text{(or if we use } 125.5 \text{ you get } \underline{\underline{25.6 \text{ minutes}}}) \text{ must be } 1 \text{ dp}
 \end{aligned}$$

3.

The histogram and frequency table show some information about the time, in hundreds of hours, that a number of similar light bulbs lasted.

Time, t hundred hours	Number of light bulbs
$0 < t \leq 8$	48
$8 < t \leq 12$	<u>32</u>
$12 < t \leq 16$	40
$16 < t \leq 20$	<u>28</u>
$20 < t \leq 30$	20



(a) Complete the frequency table and the histogram

Frequency = Area

$$8 < t \leq 12 = 4 \times 8 = \underline{32}$$

$$16 < t \leq 20 = 4 \times 7 = \underline{28}$$

$20 < t \leq 30$

width = 10, area = 20

$$\therefore \underline{\text{height / freq. density} = 20 \div 10 = \underline{2}}$$

[4]

(b) Find the estimate for the number of light bulbs that lasted between 2000 hours and 2400 hours.

$$= 4 \times 2 = \underline{8 \text{ lightbulbs}}$$

[1]

- (c) There are 56 bulbs that lasted less than y hundred hours.
Calculate an estimate for y .

48 lightbulbs lasted $0 < t \leq 8$

$\therefore$ we require 8 lightbulbs 'onto' next group. Next group contains 32 lightbulbs $\therefore \frac{8}{32} = \frac{1}{4}$ of width of interval

width = 4, $\frac{1}{4}$ of 4 = 1 $\therefore$ 1 hundred in interval
8 hundred hours + 1 hundred hour = 9 hundred hours

[3]

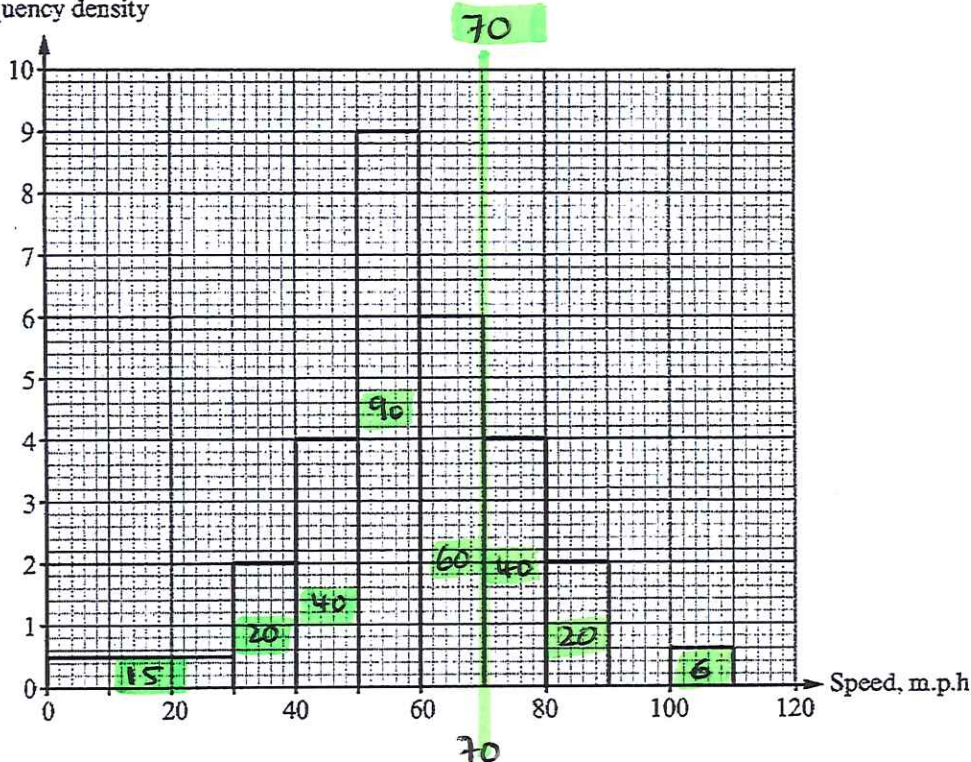
$\therefore$ 56 lightbulbs lasted less than 9 hundred hours

or $y = 9$

4.

The histogram below shows the speeds of motorists as they enter a tunnel between 1 a.m. and 2 a.m.

Frequency density



- (a) The speed limit on entering the tunnel is 70 m.p.h.
How many motorists were exceeding the speed limit on entering the tunnel?

$$(10 \times 4) + (10 \times 2) + (10 \times 0.5)$$

$$= 40 + 20 + 5 = \underline{66 \text{ Motorists}}$$

[3]

→ let $s = \text{speed}$

- (b) In order to compare the speeds of motorists between 1 a.m. and 2 a.m. with other 1 hour periods, it is decided to group the data in equal intervals of width 30 m.p.h., starting at 0 m.p.h.

Construct a histogram to display these results meeting this new requirement.

$$0 \leq s < 30 = 15$$

$$30 \leq s < 60 = 20 + 40 + 90 = 150$$

$$60 \leq s < 90 = 60 + 40 + 20 = 120$$

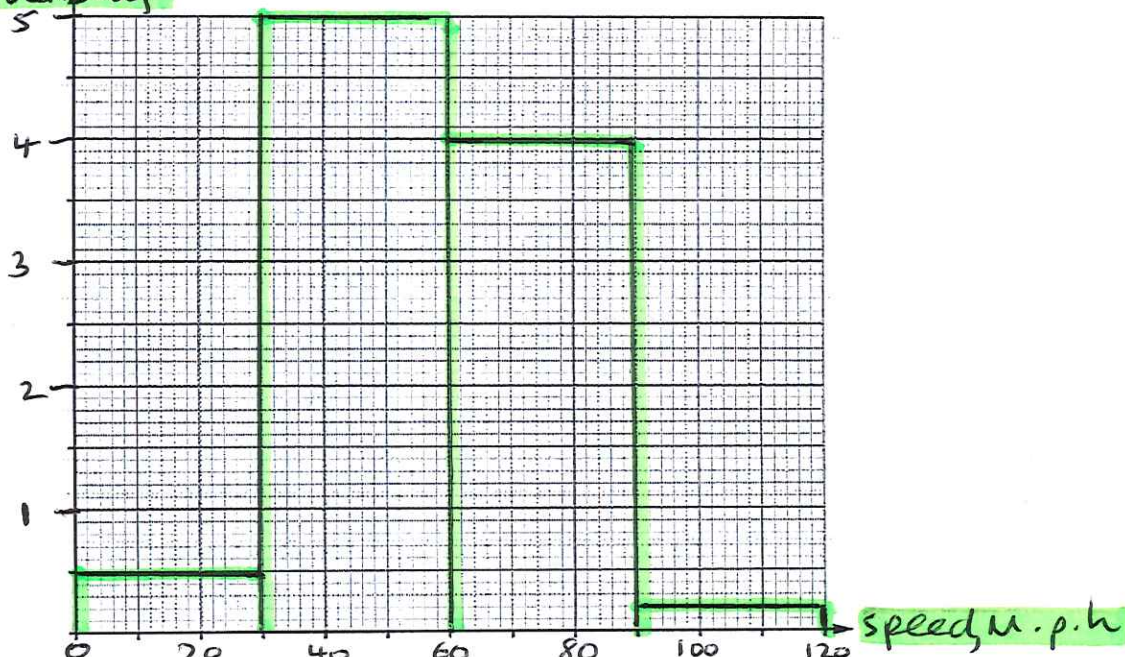
$$90 \leq s < 120 = 6$$

(all of these
come from
working out
the areas of
the bars)

You must choose a suitable scale, label your axes & there must be no gaps between your bars.

[8]

Frequency density



- (c) Given that the speed limit is 70 m.p.h., which of the two histograms is the more appropriate for the display of the data? You must give a reason for your answer.

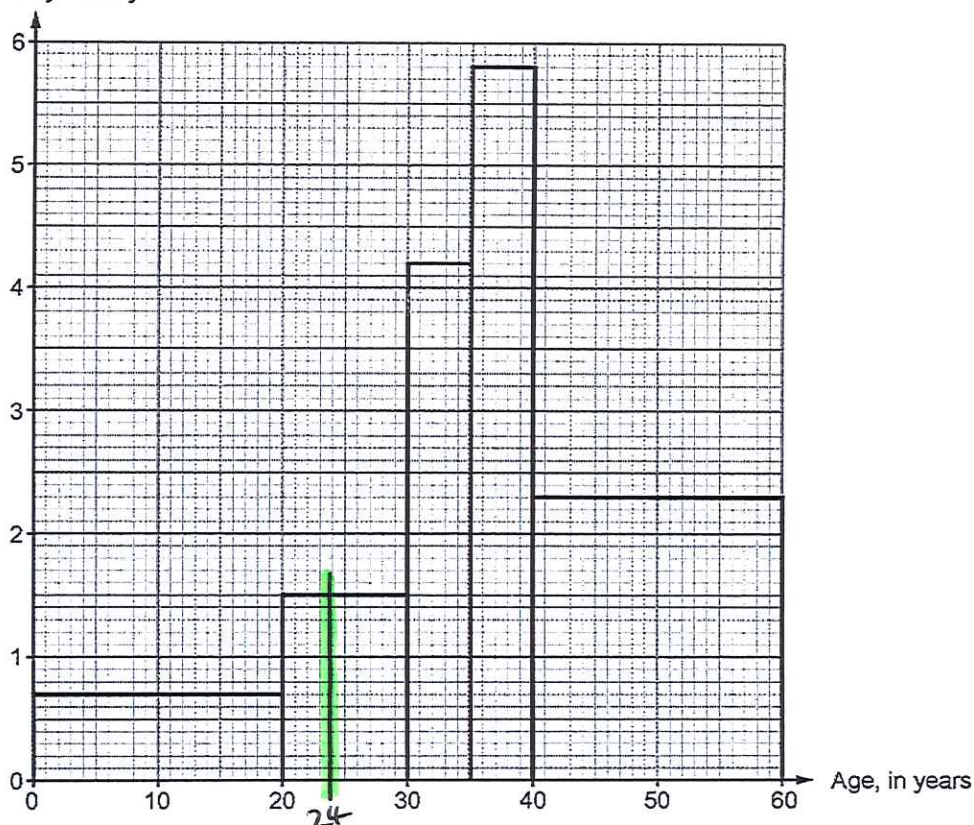
The first one is better for this speed limit because it actually shows the 70 m.p.h.

[1]

5.

The histogram below shows the ages of the people staying in a hotel one weekend.

Frequency density



(a) Use the histogram to complete the grouped frequency table below.

[2]

Age in years	$0 \leq a < 20$	$20 \leq a < 30$	$30 \leq a < 35$	$35 \leq a < 40$	$40 \leq a < 60$
Frequency	<u>14</u>	<u>15</u>	<u>21</u>	<u>29</u>	<u>46</u>

Work out the areas of all of the bars.

$$(0.7 \times 20) = 14 \quad (1.5 \times 10) = 15 \quad (4.2 \times 5) = 21$$

$$(5.8 \times 5) = 29 \quad (2.3 \times 20) = 46$$

(b) Calculate an estimate of the number of people whose ages are less than 24 years old.

[2]

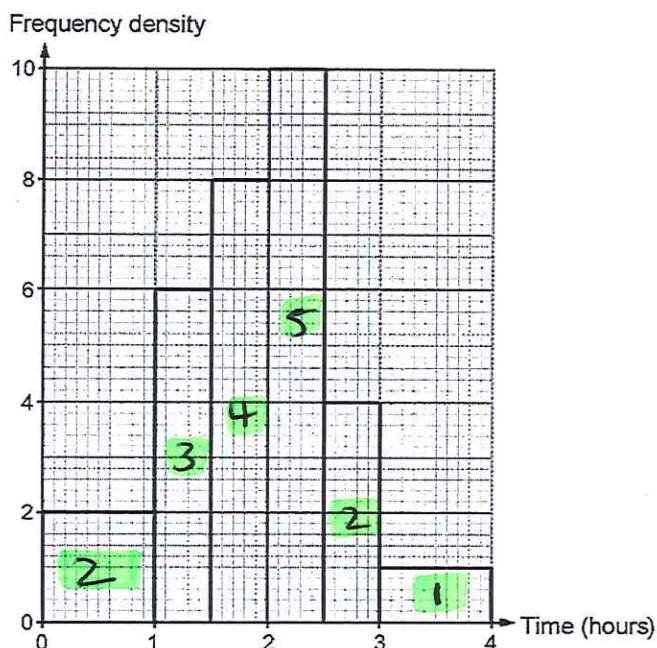
$$14 + \left(\frac{4}{10} \times 15 \right) = \underline{20 \text{ people}}$$

6.

The *Big Fish Cymru* annual fishing competition is held on the west coast of Wales. Information about last year's competition is displayed in the *Big Fish Cymru* booklet. A section of this booklet is shown below.

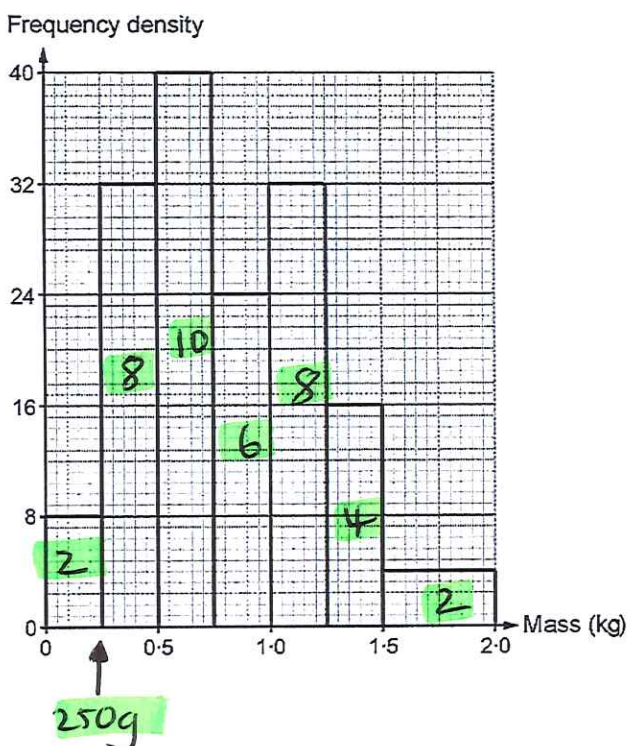
The competition organisers recorded the time taken for each angler to catch their first fish.

This is shown in the histogram on the right.



The competition organisers also recorded the mass of every fish caught.

This is shown in the histogram on the right.



- (a) Last year, how many of the fish caught had a mass of less than 250g? [1]

$$0.25 \times 8 = \underline{2 \text{ fish}}$$

- (b) Last year, the final angler to catch their first fish did so after $3\frac{1}{2}$ hours.

How many other anglers took more than 3 hours to catch their first fish? [1]

0

(because there was only 1 fish caught after 3 hours as area = $1 \times 1 = 1$)

- (c) The number of anglers taking part this year was three times as many as took part last year.

How many anglers took part in the competition this year? [4]

From first histogram, number of angles must

$$\text{be} = (1 \times 2) + (0.5 \times 6) + (0.5 \times 8) + (0.5 \times 10)$$

$$+ (0.5 \times 4) + (1 \times 1)$$

$$= 2 + 3 + 4 + 5 + 2 + 1 = \underline{17 \text{ last year}}$$

$$\therefore \text{This year} = \underline{17 \times 3 = 51}$$

Number of anglers this year was 51

- (d) The median mass of the fish caught this year was 0.9kg.

What is the difference, in kg, between the median mass of the fish caught this year and the median mass of the fish caught last year? [5]

Number of fish last year (from areas of second

$$\text{histogram}) = (0.25 \times 8) + (0.25 \times 32) + (0.25 \times 40)$$

$$+ (0.25 \times 24) + (0.25 \times 32) + (0.25 \times 16) + (0.5 \times 4)$$

$$= 2 + 8 + 10 + 6 + 8 + 4 + 2 = \underline{40 \text{ fish}}$$

$$\underline{\text{Median}} = 20^{\text{th}} \text{ fish} = \underline{0.75 \text{ kg}}$$

$$\text{Difference in mass} = 0.9 - 0.75 = 0.15$$

Difference in mass is 0.15 kg

(examiner follows through if you use 20.5^{th} fish)

(e) Approximately 10% of the anglers this year caught their first fish within 1 hour.

(i) How does this percentage compare with last year's percentage?
You must show all your working.

[3]

Last year's % within 1 hour

$$= \frac{2}{17} \times 100$$

$$= \frac{200}{17}$$

$$= 11.8\% \text{ (1dp)}$$

$$\begin{array}{r} 11.76 \\ 17 \overline{) 200.00} \end{array}$$

$$11.8\% > 10\%$$

∴ A greater percentage of the anglers caught their first fish within 1 hour last year

(ii) Do you think it is fair to compare last year's competition results with this year's competition results?
You must give a reason for your answer.

[1]

No, reason might be

- numbers taking part very different
- the competition has grown
- the conditions might not be the same
- the weather might be very different

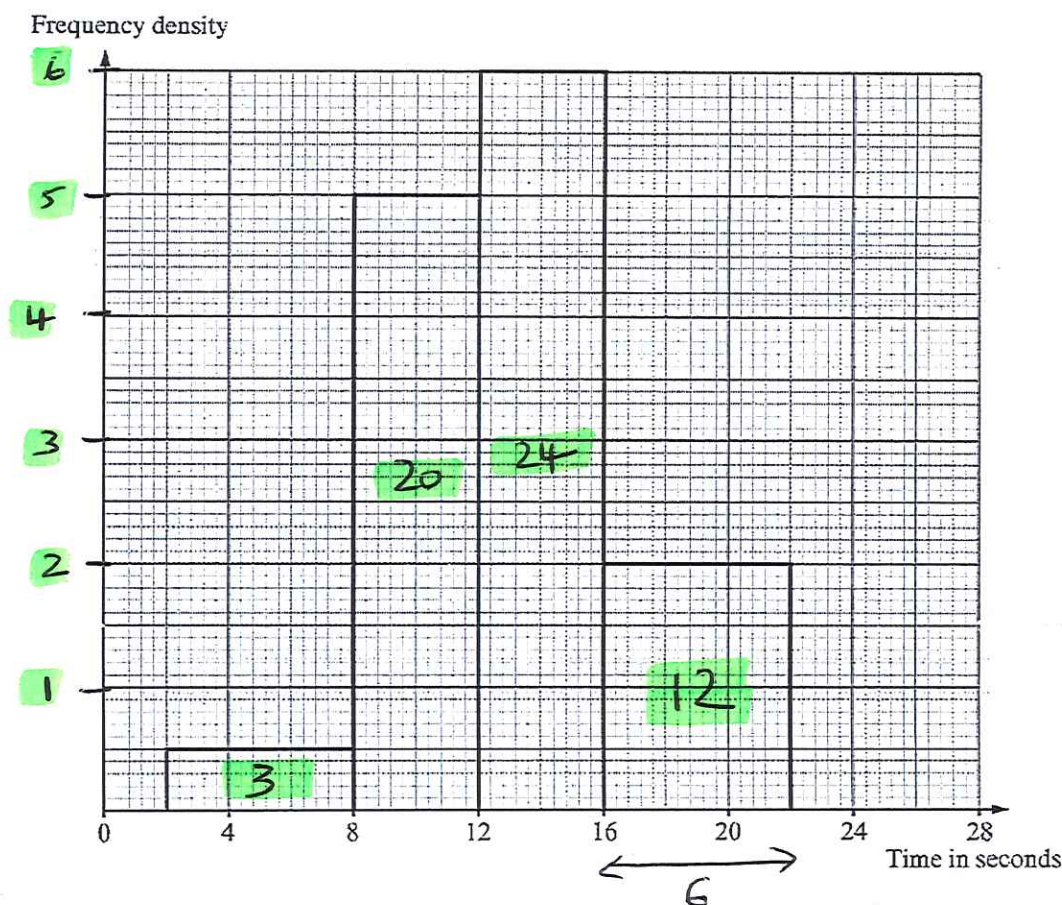
examiner awards any sensible answer

examine accepts

YES • we are comparing like with like if we use percentages / proportions

7.

- (a) As part of a quality control exercise in a supermarket, the time taken to scan 20 items was measured for each worker under the age of 40. A printout of the histogram that illustrates the results obtained is shown below.



Unfortunately, the labelling of the frequency density axis was missing from the printout. It is known that there were 12 workers under the age of 40 that took more than 16 seconds to scan the 20 items.

- (i) Complete the labelling of the scale on the frequency density axis.

$$16 < t \leq 22 = \text{area of } 12$$

$$= 6 \times 2 \leftarrow \text{gives height of } 2$$

so we can now fully label the frequency density axis.

[3]

- (ii) Calculate how many workers under the age of 40 took part in this quality control exercise.

$$\begin{aligned} & (6 \times 0.5) + (4 \times 5) + (4 \times 6) + (12) \\ &= 3 + 20 + 24 + 12 \\ &= \underline{59 \text{ workers}} \end{aligned}$$

[2]

- (iii) Calculate an estimate of the median time taken by a worker under the age 40 to scan 20 items.

$$\text{Median} = \frac{59 + 1^{\text{th}}}{2} = \underline{30^{\text{th}} \text{ worker}} \quad (\text{or } 29.5^{\text{th}})$$

23 below 12 minutes

$\therefore$ require 7 'into' $12 < t \leq 16$

$$= 12 + \frac{7}{24} \times 4$$

$$= \underline{13.17 \text{ seconds}} \quad (2dp)$$

[4]

or if use 29.5^{th}

require 6.5 into $12 < t \leq 16$

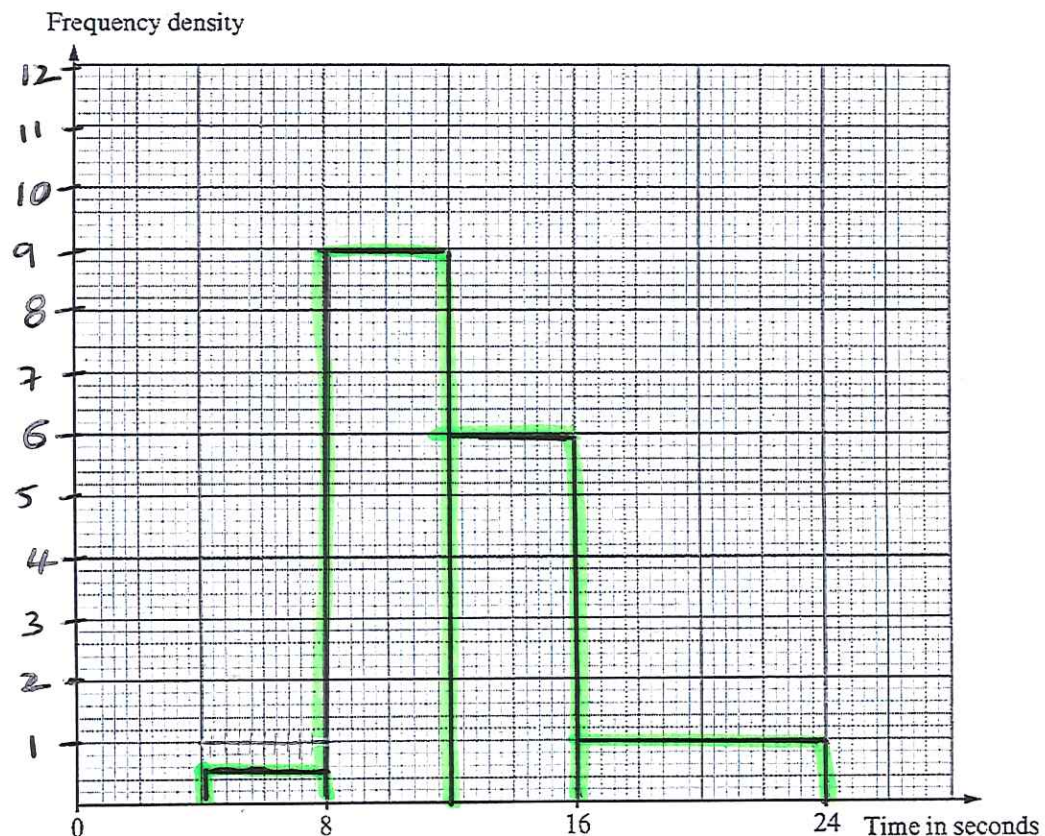
$$= 12 + \frac{6.5}{24} \times 4$$

$$= \underline{13.08 \text{ seconds}} \quad (2dp)$$

- (b) As part of the quality control exercise in a supermarket, the time taken to scan 20 items was measured for each worker aged 40 or over. The table below shows the results.

Time in seconds, t	$0 < t \leq 4$	$4 < t \leq 8$	$8 < t \leq 12$	$12 < t \leq 16$	$16 < t \leq 24$
Number of workers	0	2	36	24	8

Complete the scale on the frequency density axis and draw a histogram to illustrate the distribution on the graph paper below.



[3]

- (c) Which of the two groups of workers is, on average, quicker at scanning 20 items in the supermarket? You must give a reason for your answer.

40 and over

Median of under 40s is about 13 seconds

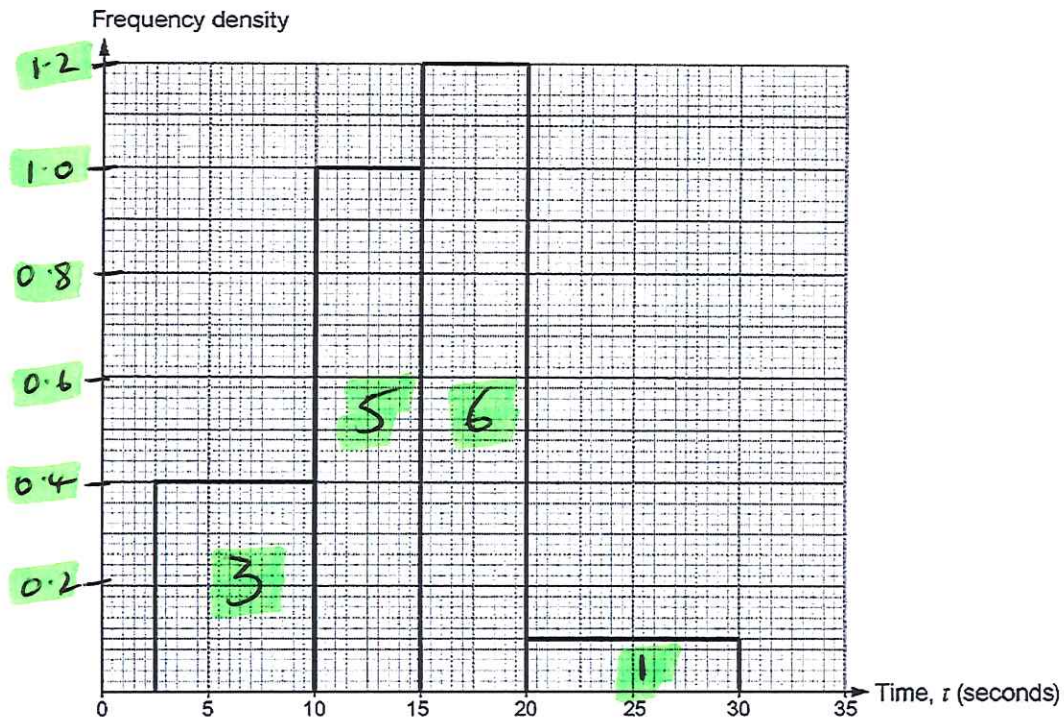
Whereas for over 40s the median will be

less than 12 seconds. (From table we

can quickly see more than half have a time less than 12 seconds. You don't need to actually work it out.)

8.

- (a) At a supermarket, the time taken for customers to scan 5 items at a self-checkout was recorded.
A histogram that illustrates the results is shown below.
Unfortunately, the scale on the frequency density axis is missing.



It is known that 6 people took between 15 and 20 seconds to scan their 5 items.

- (i) Complete the scale on the frequency density axis. [3]

15-20 seconds bar has a width of 5
 $\therefore \text{height} = 6/5 = 1.2$

- (ii) How many people were timed scanning their 5 items? [2]

Work out areas of all the bars and add

$$\begin{aligned}
 &= (5 \times 0.4) + (5 \times 1.0) + (5 \times 1.2) + (10 \times 0.1) \\
 &= 3 + 5 + 6 + 1 \\
 &= 15 \text{ people}
 \end{aligned}$$

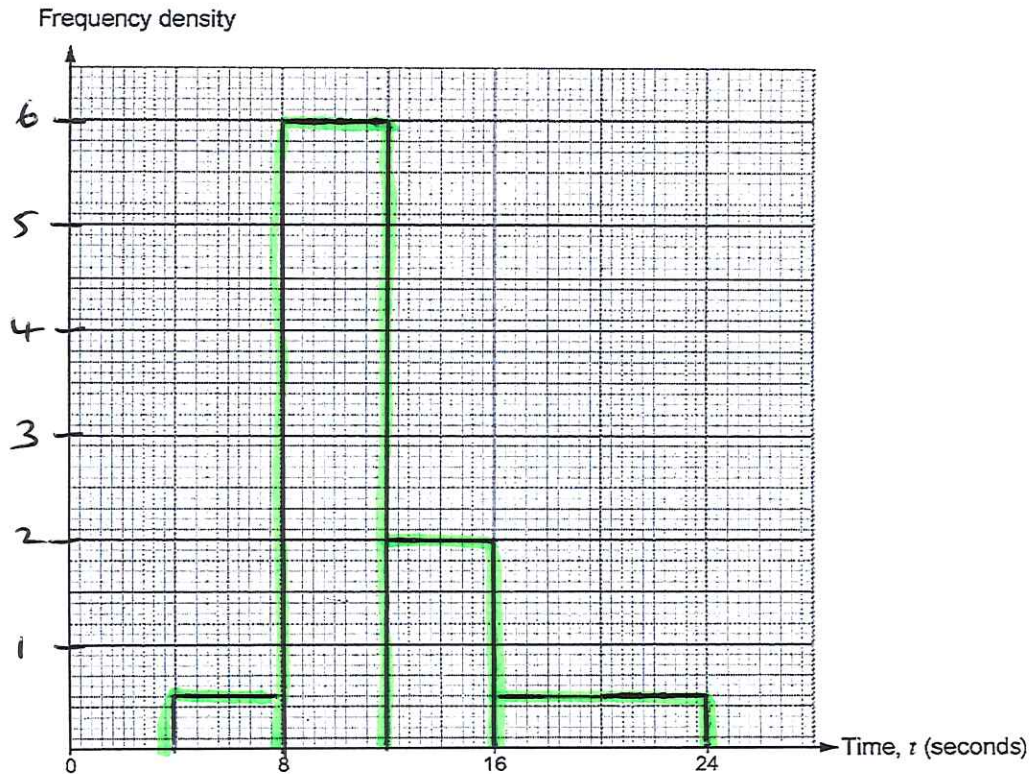
- (b) The time taken for staff to scan 5 items was also recorded.

The table below shows the results.

Time, t (seconds)	$0 < t \leq 4$	$4 < t \leq 8$	$8 < t \leq 12$	$12 < t \leq 16$	$16 < t \leq 24$
Number of staff	0	2	24	8	4

Complete the scale on the frequency density axis and draw a histogram to illustrate the distribution on the graph paper below. [3]

Biggest height = $24 \div 4 = 6$, so need frequency density to go up to 6.



- (c) On average, which of the two groups of people, customers or staff, were quicker at scanning 5 items?

Tick the appropriate box.

Customers

☐

Staff

☒

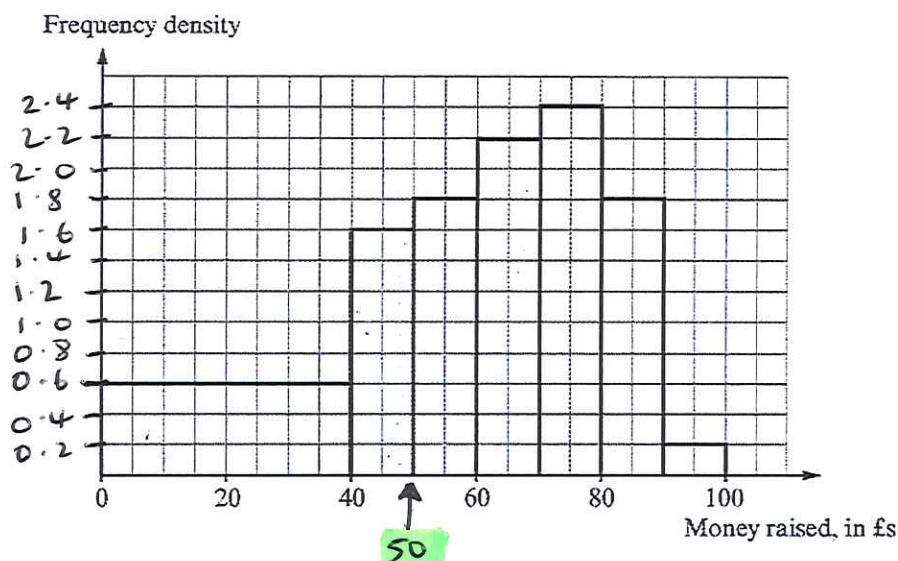
You must use the histograms to explain your answer.

[1]

Median for staff will be in the group 8 to 12 seconds. Whereas the median for the customers will be more.

9.

Jack arranged a raffle to raise money.
He has drawn a histogram to show the distribution of money raised from the raffle.



Jack has forgotten to write the scale on the vertical axis.
He knows that 40 people each raised £50 or less.
Calculate an estimate for the total money raised.

40 small squares = 40 people
(literally count the number of squares up to £50)
∴ 24 up to £40
and 16 between £40 - £50
∴ area 0-40 = 24 = 40 × height
height = 24 ÷ 40 = 0.6

Money raised (£m)	0 < m ≤ 40	40 < m ≤ 50	50 < m ≤ 60	60 < m ≤ 70
Number	24	16	18	22

[6]

70 < m ≤ 80	80 < m ≤ 90	90 < m ≤ 100
24	18	2

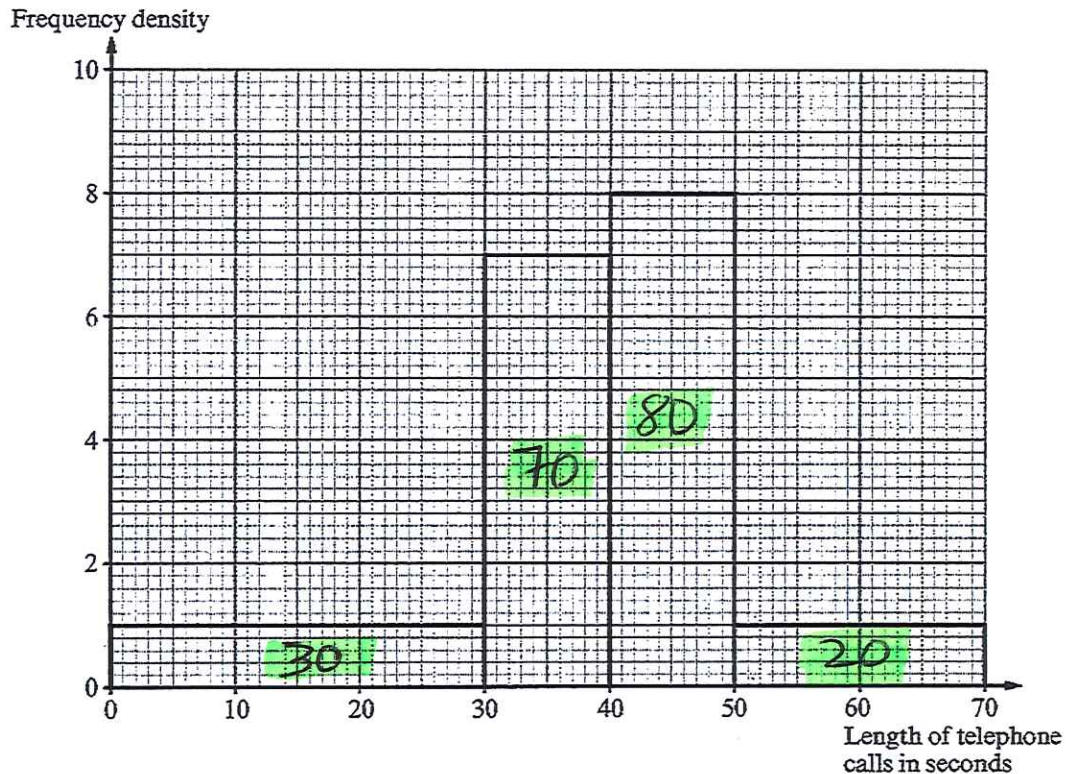
Now work out total using midpoints

$$\begin{aligned} \text{Total} &= (20 \times 24) + (45 \times 16) + (55 \times 18) + (65 \times 22) + (75 \times 24) \\ &\quad + (85 \times 18) + (95 \times 2) = \text{£}7140 \end{aligned}$$

Estimate of total raised = £7140

10.

The histogram illustrates the lengths of telephone calls made to a computer helpline during one Tuesday evening.



- (a) Calculate how many telephone calls were made to the computer helpline during the Tuesday evening.

$$(30 \times 1) + (10 \times 7) + (10 \times 8) + (20 \times 1)$$

$$= 30 + 70 + 80 + 20$$

$$= \underline{200 \text{ phone calls}}$$

[3]

- (b) Estimate the median length of a telephone call made to the computer helpline during the Tuesday evening.

$$\text{median} = \underline{100^{\text{th}}}$$

$$= \underline{40 \text{ seconds}}$$

[2]