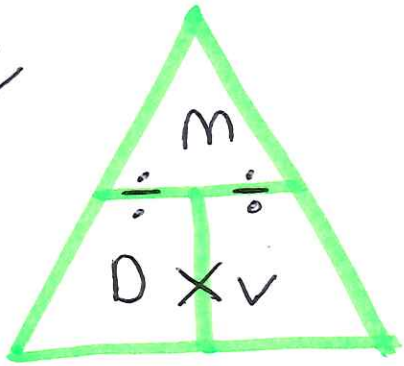
MATHSDIY

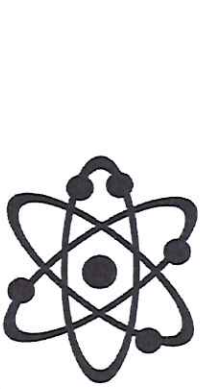
SOLUTIONS

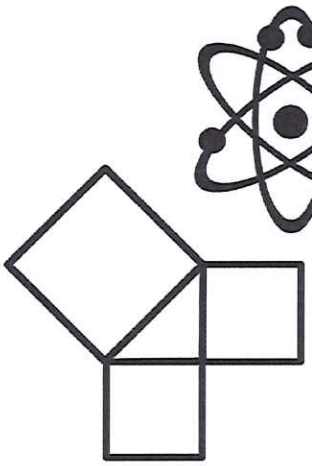
GCSE TOPIC BOOKLET
DENSITY

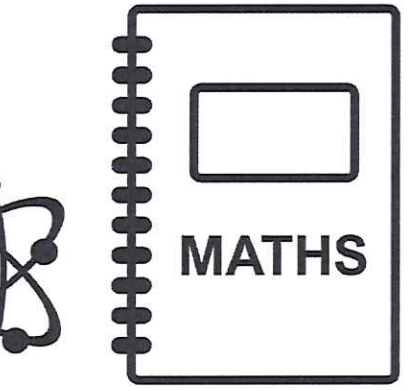
Density = Mass / Volume

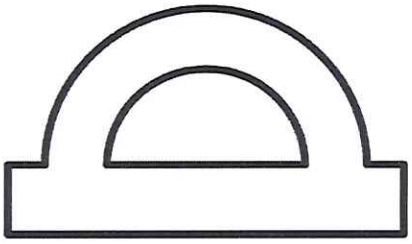
or

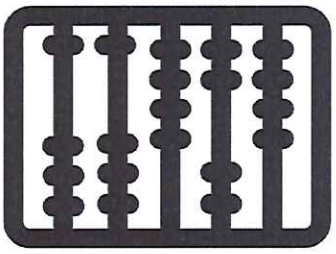


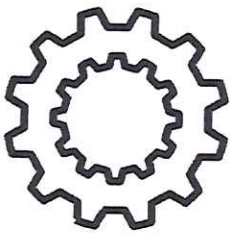


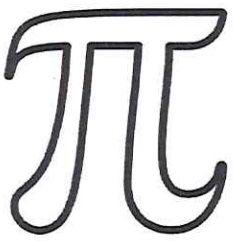


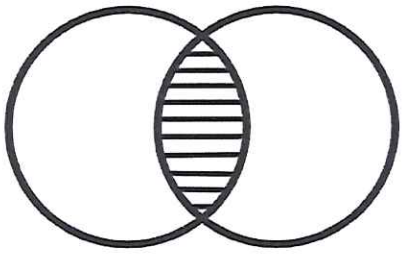


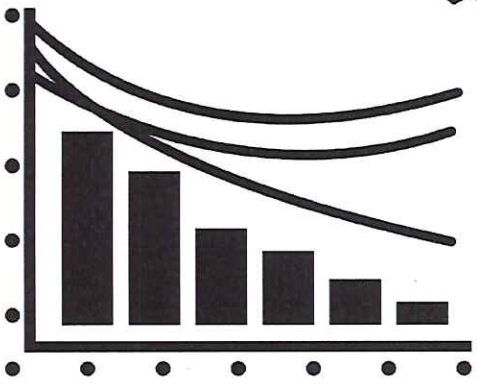


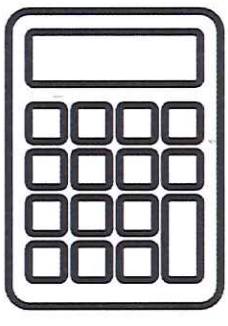


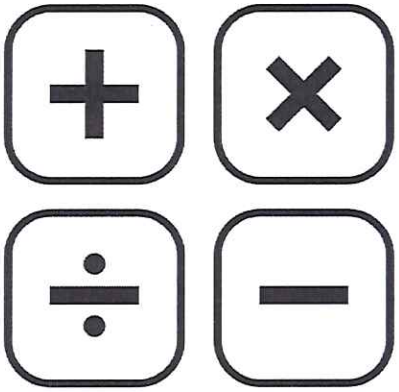












1.

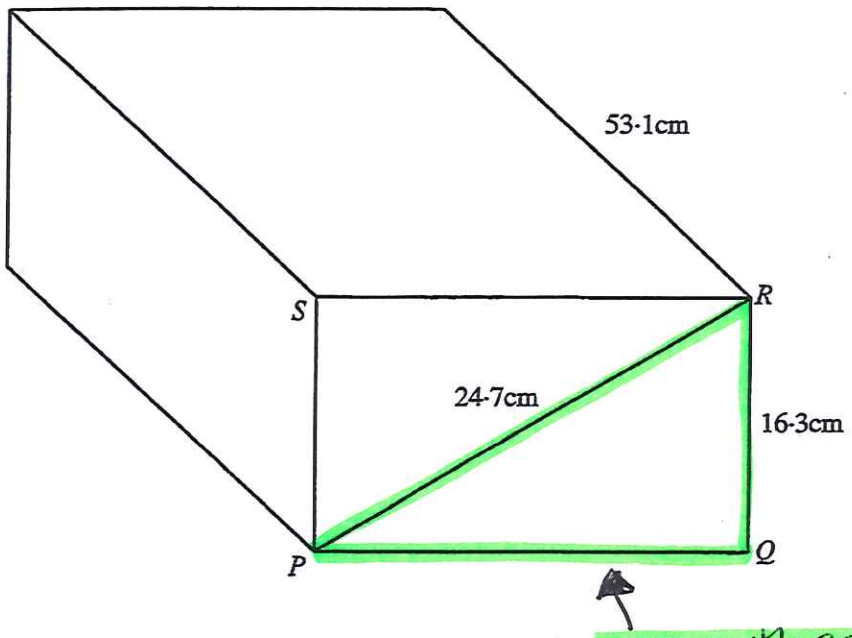


Diagram not drawn to scale.

use pythagoras

The diagram shows a cuboid of length 53·1 cm. The cross-section, $PQRS$, is such that $PR = 24{\cdot}7$ cm and $QR = 16{\cdot}3$ cm.

(a) Calculate the length of PQ.

$PQ^2 = PR^2 - RQ^2$

$PQ^2 = 24{\cdot}7^2 - 16{\cdot}3^2$

$PQ^2 = 344{\cdot}4$

$PQ = \sqrt{344{\cdot}4} = 18{\cdot}558\ldots$

$PQ = 18{\cdot}56$ cm (2dp)

[3]

(b) The density of the material from which the cuboid is made is 4·3 g/cm^3. Calculate the mass of the cuboid in kilograms. (TIP:- must find volume first)

$V = L \times W \times H$

$V = 18{\cdot}56 \times 16{\cdot}3 \times 53{\cdot}1 = 16064{\cdot}2368$

mass = density × volume

mass $= 4{\cdot}3 \times 16064{\cdot}2368 = 69076{\cdot}21\ldots 9$

Require mass in kg ∴ ÷1000

[3]

Mass = 69·08 kg

(or 69·07 kg if you didn't round PQ)

2.

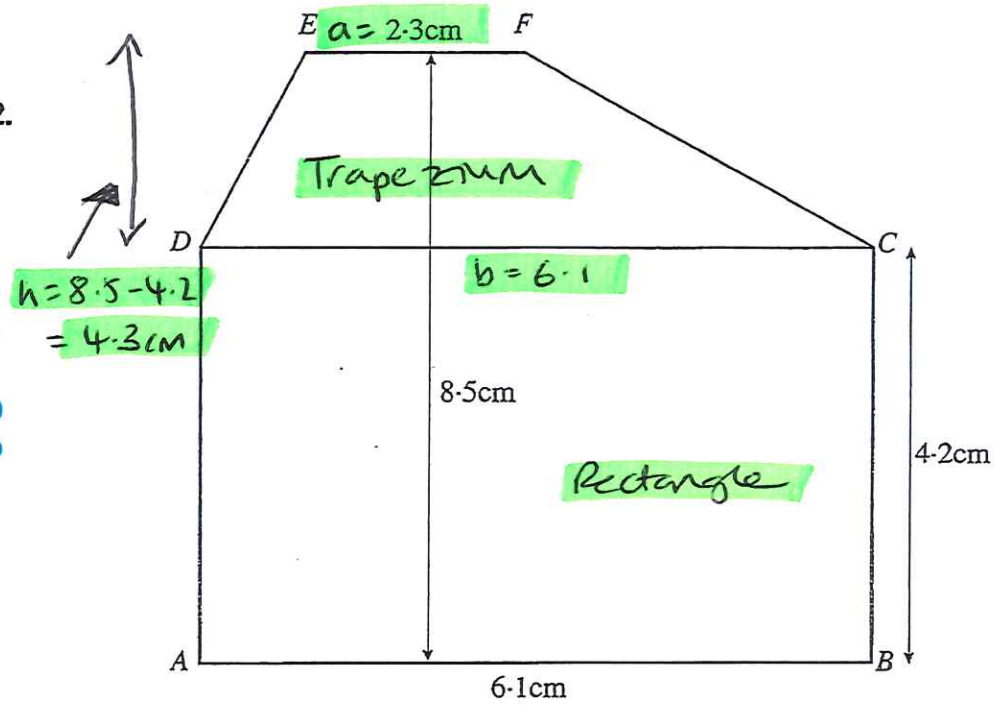


Diagram not drawn to scale.

ABCFED represents the uniform cross-section of a solid block of material. *ABCD* is a rectangle in which *AB* = 6·1 cm and *BC* = 4·2 cm. *EF* is of length 2·3 cm and is parallel to *AB*. The distance between *EF* and *AB* is 8·5 cm.

(a) Calculate the area of cross-section of the block.

Area of rectangle $= L \times W = 6.1 \times 4.2 = 25.62\text{cm}^2$

Area of trapezium $= \frac{1}{2}(a+b) \times h = \frac{1}{2}(2.3+6.1) \times 4.3$

$= 18.06\text{cm}^2$

Area of cross section $= 25.62 + 18.06 = 43.68\text{cm}^2$

[3]

(b) The block has this uniform cross-section along its length of 12·6 cm and has a mass of 2 kg. Calculate the density, in g/cm^3, of the material from which the block is made.

2000 g

Volume = Cross sectional area × length

Volume $= 43.68 \times 12.6 = 550.368\text{cm}^3$

Density = mass ÷ volume

$= 2000 \div 550.368$

$= 3.633\ldots$

$= 3.6\text{ g/cm}^3$ (1 dp)

[4]

MATHSDIY

3. (a) A ski bag has a label which states that the fabric used to make the bag weighs 920 g per m^2.
A rectangular piece of this fabric weighs 0·46 kg.
The piece of fabric is 0·4 m wide.
Calculate the length of this piece of fabric in metres. [4]

mass

area density

Need Area Given mass = 0.46kg = 0.46 × 1000
= 460g

Given density of fabric = 920g/m^2

Area must be * 460 ÷ 920 = 0.5m^2

L × W = 0.5

L × 0.4 = 0.5

$L = \frac{0.5}{0.4} = 1.25m$ Length = 1.25m

(b)

	Old skis	Person	New skis
Paulo is 177 cm tall. His old skis are 173 cm long. His new skis are 166 cm long.			
Layla is 163 cm tall. Her old skis are 160 cm long. Her new skis are 153 cm long.			

* In this question we are using 'area density' which in this particular example is measured in grams per square metre. So,

$$\text{area density} = \frac{\text{mass (g)}}{\text{area } (m^2)}$$

$$\therefore \text{area} = \frac{\text{mass}}{\text{area density}}$$

Paulo says,

'In proportion to our heights, my new skis are shorter for me than Layla's new skis are for her.'

Layla says,

'Paulo can't be right!
We have both gone for 7 cm shorter skis than our old ones.'

Is Paulo correct?
You must show your calculations and give a reason for your answer. [4]

Conside what percentage of their heights the skis are.

Paulo :- New skis are $\frac{166}{177} \times 100$

$= 93.785\%$ of his height

Layla :- New skis are $\frac{153}{163} \times 100 =$

$= 93.865\%$ of he height

Paulo is right, Paulo's skis are proportionally just less than Layla's skis.

4.

This is similar to q3 an area density question

Ben and Catrin both buy new fleece jackets.

The label inside Ben's jacket says:

Area density = mass / area

area = mass / area density

Astra Jacket
90% Polyester, 10% Elastane
Soft fleece fabric 180g/m²

The label inside Catrin's jacket says:

Snug Jacket
80% Polyester, 20% Elastane
Stretchy fleece fabric 140g/m²

Ben and Catrin know the following facts about their jackets.

- Ben's jacket weighs 234g.
- Catrin's jacket is made with 8% less fleece fabric than Ben's jacket.

(a) How much does Catrin's *Snug Jacket* weigh?
You must show your working. [4]

Area

Fabric in Ben's fleece = 234 ÷ 180
= 1·3m²

Fabric in Catlin's fleece = 8% less
= 0·92 × 1·3
= 1·196m²

(Mass) Weight = 140g/m² × 1·196m²
= 167·44g

Least $139\ g/m^2$
Greatest $141\ g/m^2$

(b) The density of the fleece fabric in *Snug Jackets* is actually $140\ g/m^2$, correct to the nearest $2\ g/m^2$.
This fabric is sold to the makers of the jackets on cardboard rolls.
A cardboard roll with fleece fabric on it weighs 4·5kg. → 4500g
The empty cardboard roll weighs 360g.

Complete the following sentence by inserting a correct value, accurate to 4 significant figures.

The roll contains at least ..... m^2 of *Snug Jacket* fleece fabric.

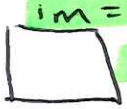
You must show your working. [6]

(mass) Fabric weighs $4500 - 360 = 4140g$

(area) Amount of fabric $= 4140 \div 141 = 29.3617\ldots$
$= 29.36\ m^2$ (4sf)

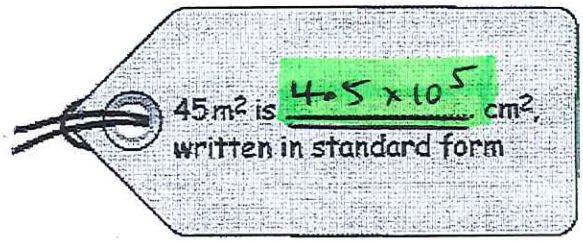
(we divide by 141 because we want to consider the worst case scenario)

(c) A different roll contains $45\ m^2$ of fleece fabric.
This roll of fleece fabric has a label attached giving the area of fabric in cm^2, written in standard form.
Complete the label below.
You must show your working. [2]

$1\ m^2 =$  $1m = 100cm$ $1m = 100cm$

$= 100 \times 100 = 10{,}000\ cm^2$

$45\ m^2 = 45 \times 10{,}000 = 450{,}000\ cm^2 = 4.5 \times 10^5\ cm^2$



5. a)

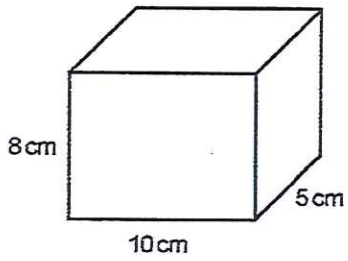


Diagram not drawn to scale

A cuboid made of metal has dimensions 10 cm, 8 cm and 5 cm. The mass of the cuboid is 1·1 kg.
Calculate the density of the metal. State the units of your answer. [4]

Find Volume first

$V = L \times W \times H$

$= 10 \times 5 \times 8 = 400 cm^3$

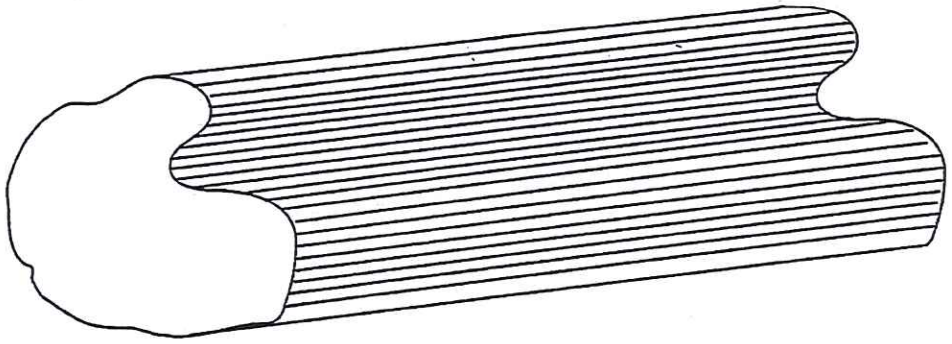
Mass = 1·1 kg = 1100 g

Density = Mass ÷ Volume

$= 1100 \div 400$

$= 2{\cdot}75 \text{ g/cm}^3$ (or 0.00275 kg/cm^3 or 2750 kg/m^3)

b)



The diagram represents a prism with an uniform cross-section of area 78 cm^2.
The prism is 54 cm long and has a mass of 19·6 kg.
Find the density, in g/cm^3, of the material from which the prism has been made.

Volume of prism = Area of cross section × length

$= 78 \times 54$

$= 4212 cm^3$

Mass of prism = $19{\cdot}6 \times 1000 = 19600$ g

Density of prism = Mass ÷ Volume

$= 19600 \div 4212$

$= 4{\cdot}65 \text{ g/cm}^3$ (2dp) [3]

6. The diagram below shows the uniform cross-section of a metal beam.

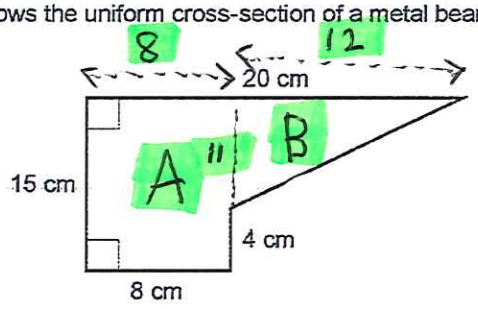


Diagram not drawn to scale

(a) *You will be assessed on the quality of your written communication in this part of the question.*

Calculate the area of the cross-section.
You must show your working. [6]

Area A = L × W = 15 × 8 = 120 cm²

Area B = ½(B × H) = ½(12 × 11) = 66 cm²

Area of cross section = 120 + 66
= 186 cm²

(b) The length of the metal beam is 2m and its mass is 108kg. Calculate the density of the metal, giving your answer in g/cm³. [4]

200cm

108 × 1000
= 108000g

Must find Volume of prism first

Volume of prism = Area of cross section × length
= 186 × 200 = 37200 cm³

Density = Mass / Volume = 108000 / 37200 = 2.90322...

= 2.90 g/cm³ (1dp)

7.

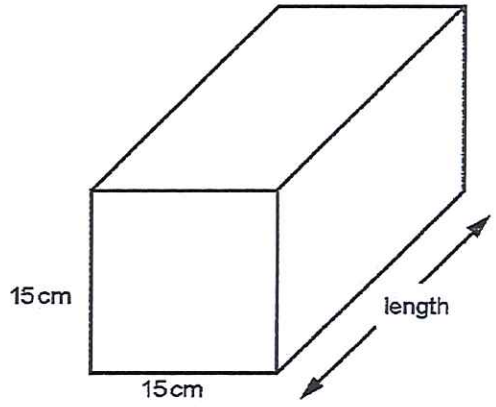


Diagram not drawn to scale

The solid block shown above is made from a metal that has a density of 2·7 g/cm³.
The volume of the solid block is 40500 cm³.

A hole is drilled through the entire length of the block.
The hole has a cross-sectional area of 25 cm².
Calculate the mass of the block that remains. [6]

Strategy ① Work out volume of whole first

∴ Need length of block

$V = L \times W \times H$

$40,500 = L \times 15 \times 15$

$40,500 \div 225 = L$ length = 180 cm

∴ Volume of Hole = cross sectional area × length

$= 25 \times 180 = 4500\ cm^2$

② Now, work out volume remaining

volume remaining = 40,500 − 4500

$= 36,000\ cm^3$

③ Finally, mass remaining = density × volume

$= 2.7 \times 36,000$

$= 97,200\ g$

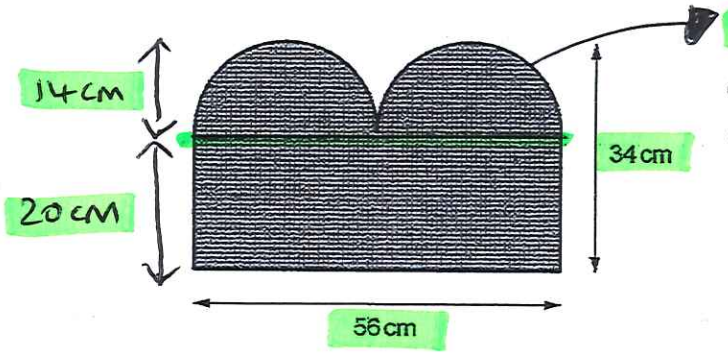
(or 97.2 kg)

8.

A beanie hat is to be made from a piece of fabric.
One square metre of this fabric weighs 420 g.

$420\ g/m^2$ = area density

The diagram below shows the pattern for the beanie hat, consisting of two identical semi-circles attached to a rectangle.



Two semicircles can be thought of as one circle of radius 14 cm

Diagram not drawn to scale

How much will the fabric in this beanie hat weigh?
Give your answer in grams. [7]

(1) Need to work out area of hat

Area of rectangle = $L \times W = 56 \times 20 = 1120\ cm^2$

Area of circle = $\pi r^2 = \pi \times 14^2 = 615.752\ cm^2$

Area of hat = $1120 + 615.752 = 1735.752\ cm^2$

(2) Need to work out weight (mass)

$1m = 100cm$

Need to change $cm^2 \Rightarrow m^2$ $\boxed{1m^2}$ ~~$1m = 100cm$~~

$1m^2 = 100cm \times 100cm = 10000cm^2$

$\therefore$ Area of hat = $1735.752 \div 10000$

$= 0.1736\ m^2$ (4dp)

(mass) Weight = area $\times$ area density

$= 0.1736 \times 420$

$= 72.9\ g$ (1dp)