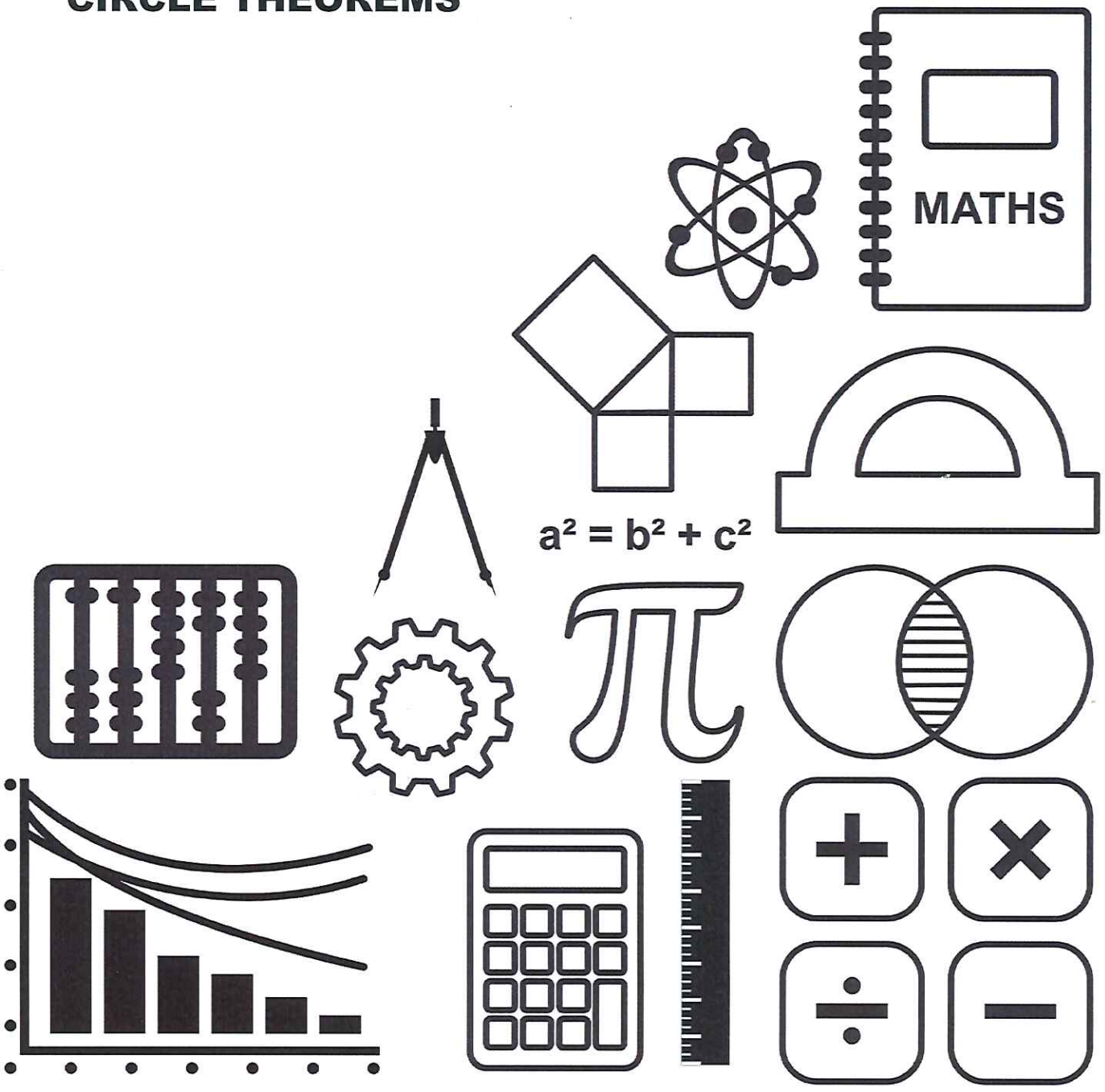
MATHSDIY

SOLUTIONS

GCSE TOPIC BOOKLET
CIRCLE THEOREMS



1.

(a) The points A, B and C lie on the circumference of a circle, centre O.

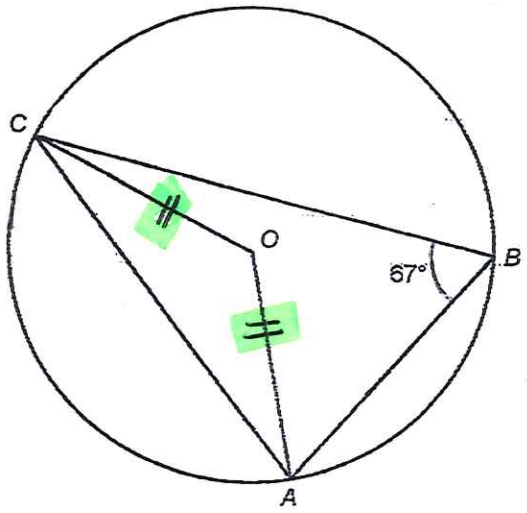


Diagram not drawn to scale

Find the size of $O\hat{A}C$. [3]

$C\hat{O}A = 134°$ (angle at centre = 2x angle circumference)

ΔOCA is isosceles, 2 equal radii

$O\hat{A}C = (180 - 134) \div 2$ $O\hat{A}C = 23°$

(b) The points D, E and F lie on the circumference of another circle. GH is a tangent to the circle at F.

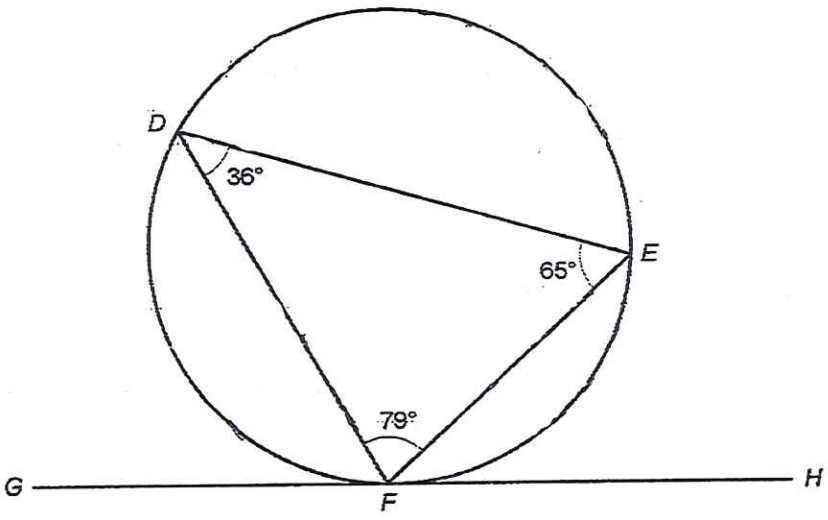


Diagram not drawn to scale

Write down the size of $E\hat{F}H$, giving a reason for your answer. [2]

$E\hat{F}H =$ 360 °

Reason: Alternate segment theorem

2.

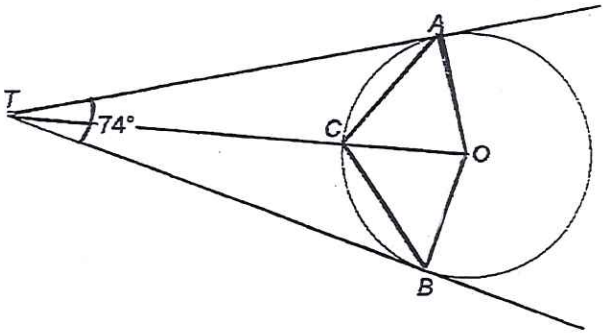


Diagram not drawn to scale

The diagram shows a circle with centre O.

AT and *BT* are tangents to the circle and $A\hat{T}B = 74°$.

(a) Calculate the size of each of the following angles.

(i) $A\hat{O}T$ [2]

$O\hat{A}T = 90°$ (angle between tangent and radius = 90°)

$T\hat{O}A = 180 - (90 + 74)$

$T\hat{O}A = 180 - 127$

$T\hat{O}A = 53°$

(ii) $O\hat{B}C$ $T\hat{O}A = T\hat{O}B = 53$ [2]

$OC = OB$ (equal radii)

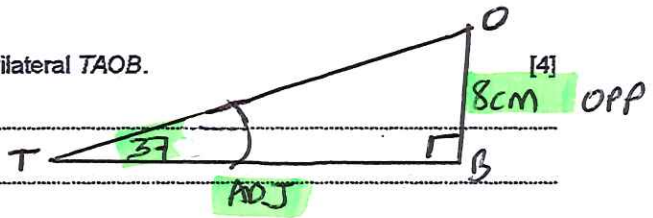
$O\hat{B}C = (180 - 53) \div 2$

$O\hat{B}C = 63.5°$

(iii) $A\hat{C}B$ $O\hat{B}C = O\hat{C}B = O\hat{C}A = 63.5°$ [1]

$A\hat{C}B = O\hat{C}B + A\hat{C}O = 2 \times 63.5 = 127°$

(b) The radius of the circle is 8cm.
Calculate the perimeter of the quadrilateral *TAOB*. [4]



$\tan 37 = \frac{8}{TB}$

$TB = 8 \div \tan 37$ $TB = 10.62cm$

Perimeter $TAOB = 10.62 + 8 + 10.62 + 8$

$= 37.2$ cm (1dp)

3.

The points P, Q and R lie on the circumference of a circle, centre O.
PQ is a diameter of the circle.
The straight line ARB is a tangent to the circle.

$\hat{QRB} = x$, where x is measured in degrees.

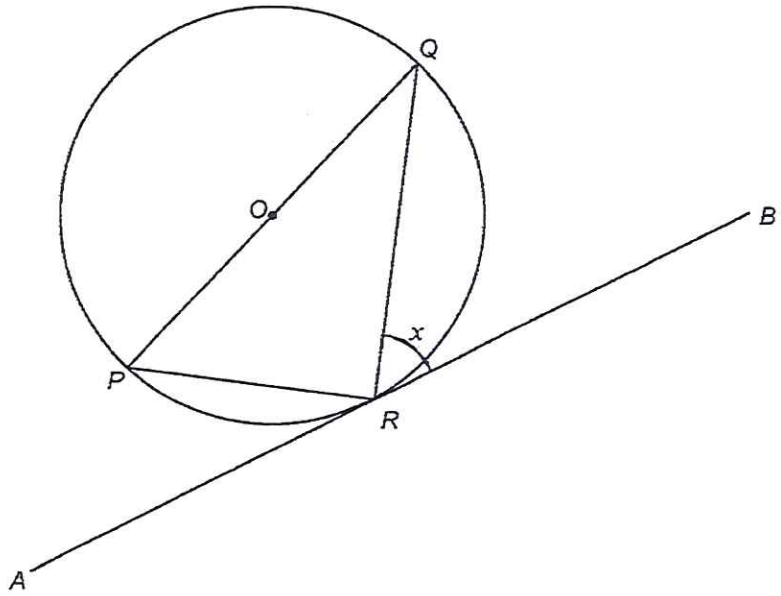


Diagram not drawn to scale

Calculate the size of $\hat{PQR}$ in terms of x.
You must give a reason for each step of your solution. [4]

$\hat{QPR} = x$ alternate segment theorem

$\hat{QRP} = 90°$ angle in a semi circle is a right angle

$\hat{PQR} = 180 - 90 - x$ angles in a triangle total 180°

$\hat{PQR} = 90 - x$

4.

The diagram shows a circle with centre O.
The straight lines AC and CE are tangents to the circle at B and D respectively.
$B\hat{F}D = 78°$.

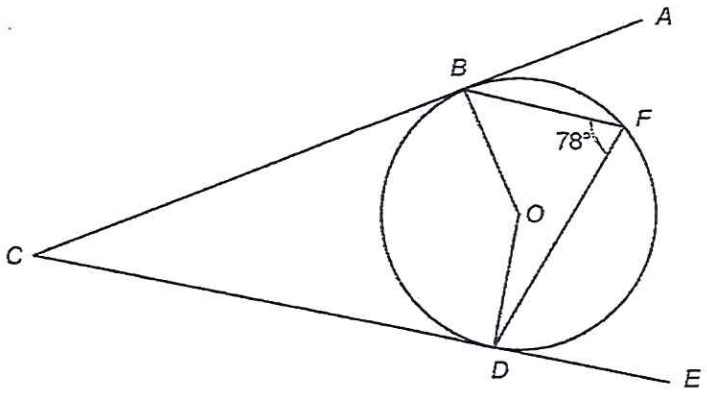


Diagram not drawn to scale

Find the size of $B\hat{C}D$. You **must** give reasons in your solution. [4]

$B\hat{O}D = 156°$ angle at the centre is twice the angle at the circumference

$O\hat{B}C = O\hat{D}C = 90°$ angle between tangent and radius is a right angle

$\therefore B\hat{C}D = 360° - (90 + 90 + 156)$

$B\hat{C}D = 360° - 336°$

$B\hat{C}D = 24°$

5.

The points A, B, C and D lie on the circumference of a circle, centre O.
EF is a tangent to the circle at C.
$AB = AC$.
$B\hat{C}E = 38°$ and $A\hat{C}D = 41°$.

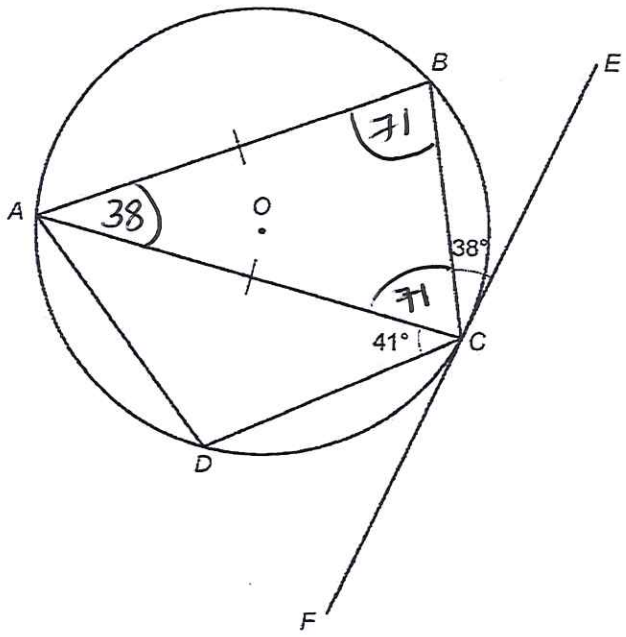


Diagram not drawn to scale

Write down the size of

(a) $B\hat{A}C$ 38° (alternate segment theorem) [1]

(b) $A\hat{B}C$ (180 − 38) ÷ 2 = 71° (isosceles triangle) [1]

(c) $A\hat{D}C$ [1]

= 180 − 71 = 109° (opposite angles in a cyclic quadrilateral add upto 180°)

(d) $C\hat{O}B$ = 76° (angle at centre = 2× angle at circumference) [1]

6. The points A and B lie on the circumference of a circle with centre O.
The straight lines PAQ and RBQ are tangents to the circle.

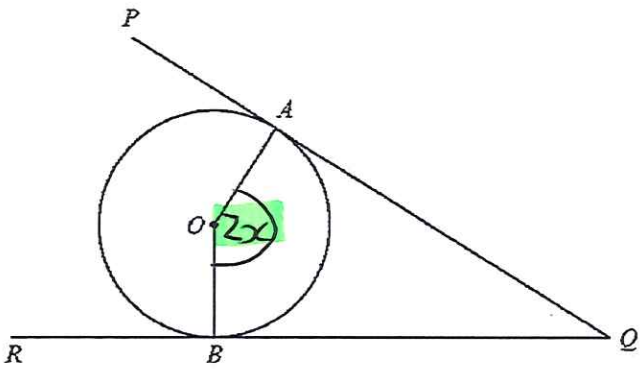


Diagram not drawn to scale

You are given that $A\hat{Q}B = 2x$, where x is measured in degrees.

Write down the size of $A\hat{O}Q$ in terms of x.
Give reasons in your answer.

$O\hat{B}Q$ and $O\hat{A}Q = 90°$
angle between tangent & radius is a right angle

Angles in a quadrilateral total 360°

$A\hat{Q}B = 360 - (90 + 90 + 2x)$

$A\hat{Q}B = 180 - 2x$

[4]

$A\hat{Q}O = \frac{1}{2} A\hat{Q}B$ (symmetry)

$A\hat{Q}O = 90 - x$

7.

(a) The diagram shows a circle with centre O and a tangent *TAP*.
The points *A*, *B* and *C* lie on the circumference of the circle.

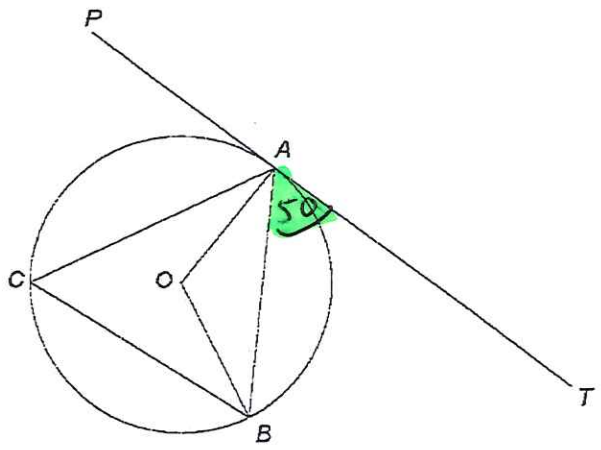


Diagram not drawn to scale

Given that $\hat{TAB} = 50°$, calculate the reflex angle $\hat{AOB}$.
You must show all your working. [3]

$A\hat{C}B = 50°$ (Alternate segment theorem)

$A\hat{O}B = 100°$ (Angle at centre = 2 x angle circumference)

Reflex $A\hat{O}B = 260°$ (angles at a point)

(b) The points B, C, D and E lie on the circumference of another circle.
The point A lies outside the circle.
ABE and ACD are straight lines.

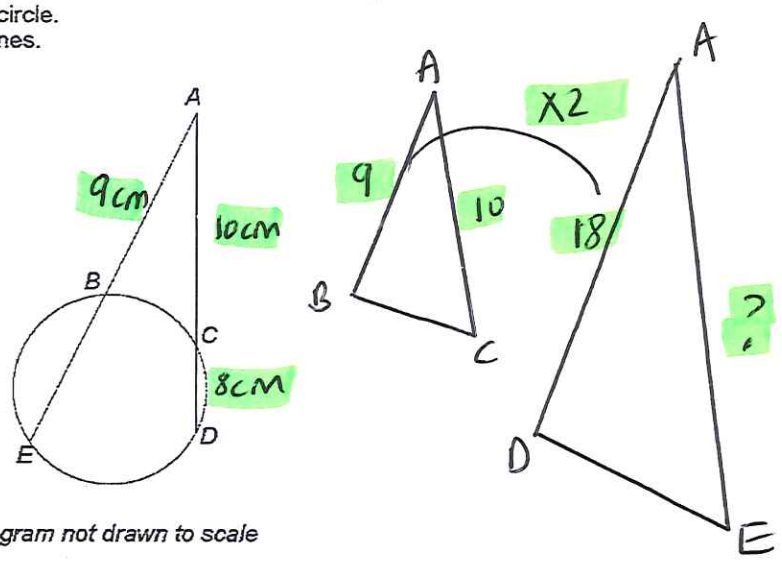


Diagram not drawn to scale

Given that $AB = 9$cm, $AC = 10$cm and $CD = 8$cm, calculate the length of BE. [2]

$9 \times 2 = 18 \quad 10 \times 2 = 20 \quad AE = 20\text{cm}$

$BE = AE - AB$

$BE = 20 - 9$

$BE = 11\text{cm}$

8.

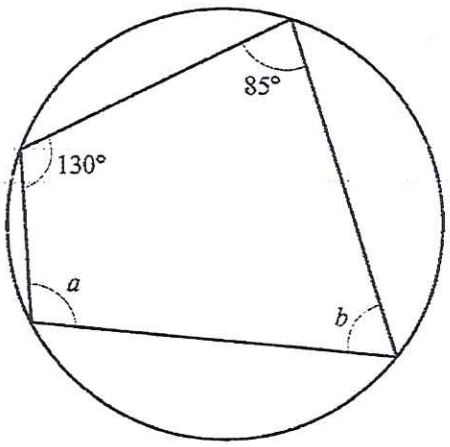


Diagram not drawn to scale

Giving a reason for your answers, calculate the size of the angles marked a and b in the diagram.

$a = 180 - 85 \quad a = 95°$
$b = 180 - 130 \quad b = 50°$
opposite angles of a cyclic quadrilateral total 180°

$a = 95°$ $b = 50°$ [3]

9.

The points A, B, C and D lie on the circumference of a circle, centre O.

AE and CE are tangents to the circle.

$C\hat{D}A = 108°$.

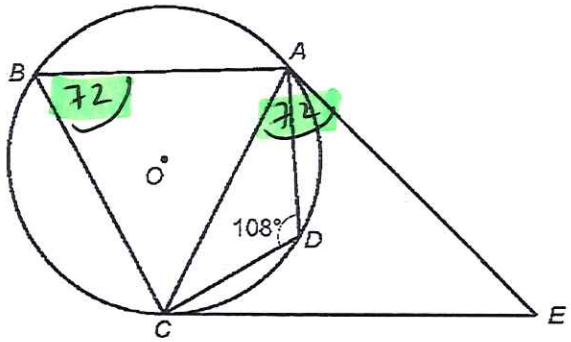


Diagram not drawn to scale

Giving **reasons** for your answers, find

(a) the size of $A\hat{B}C$, [2]

$72°$ $(180 - 108)$

opposite angles in a cyclic quadrilateral total $180°$

(b) the size of $C\hat{E}A$. [3]

$C\hat{A}E = 72°$ (alternate segment theorem)

$A\hat{C}E = 72°$

(equal tangents, isosceles triangle)

$C\hat{E}A = 180 - (72 + 72)$ (angle sum of a triangle)

$= 180 - 144$

$C\hat{E}A = 36°$

10. The points A, B, C and D lie on the circumference of a circle, centre O.
Triangle ABC is **equilateral**.
EF is a tangent to the circle at A.
$A\hat{C}D = 46°$.

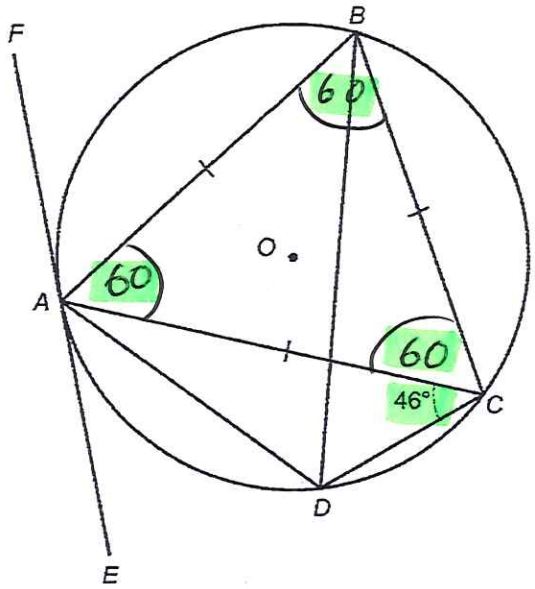


Diagram not drawn to scale

Write down the size of

(a) $A\hat{B}D$ 46° (angles in the same segment) [1]

(b) $C\hat{A}E$ 60° (alternate segment theorem) [1]

(c) $A\hat{O}B$ 120° (angle at centre is double angle at circumference) [1]

(d) $B\hat{A}D$ 74 $A\hat{B}O = A\hat{C}D$ (angles in the same segment) $= 46°$ [1]

$D\hat{B}C = D\hat{A}C = 14°$ (angles in the same segment)

$B\hat{A}D = B\hat{A}C + D\hat{A}C = 60 + 14 = 74°$

11.

The three points A, B and C lie on the circumference of a circle centre O.
The tangent XAY touches the circle at A.

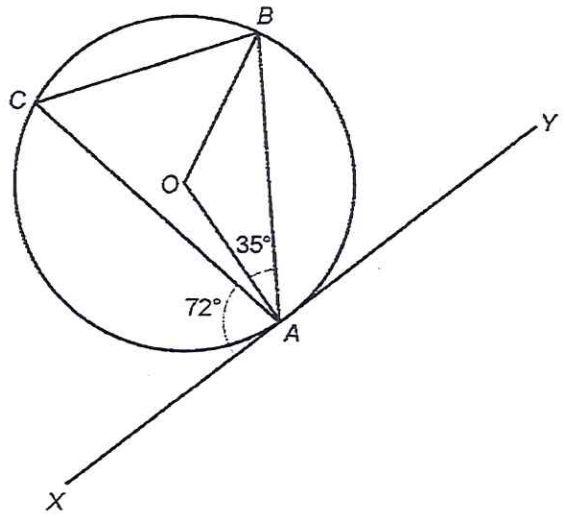


Diagram not drawn to scale

Find each of the following angles.
Give reasons for your answers.

(a) $C\hat{B}O$ $C\hat{B}A = 72°$ alternate segment theorem [2]

$O\hat{B}A = 35°$ isosceles triangle

$C\hat{B}O = 72 - 35$ $C\hat{B}O = 37°$

(b) $B\hat{C}A$ $B\hat{O}A = 180 - (2 \times 35) = 110°$ isosceles triangle [2]

$B\hat{C}A = 110/2 = 55°$ angle at centre = 2x angle at circumference

12.

The points A, B, C and D lie on the circumference of the circle with centre O.
$B\hat{C}D = x$, where x is measured in degrees.

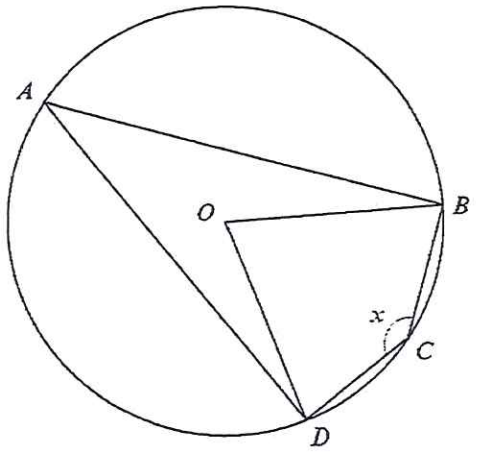


Diagram not drawn to scale

Show, giving reasons in your answer, that the size of $D\hat{O}B$ in degrees is $360 - 2x$.

$D\hat{A}B = 180 - x$ opposite angles in a cyclic quadrilateral add upto 180°

$D\hat{O}B = 2 \times D\hat{A}B$ angle at centre = 2x angle at circumference

$D\hat{O}B = 2x(180 - x)$

$D\hat{O}B = 360 - 2x$

[4]

13.

The diagram shows a circle with $BC = 30\,\text{cm}$, $AB = 50\,\text{cm}$ and $CD = 25\,\text{cm}$.

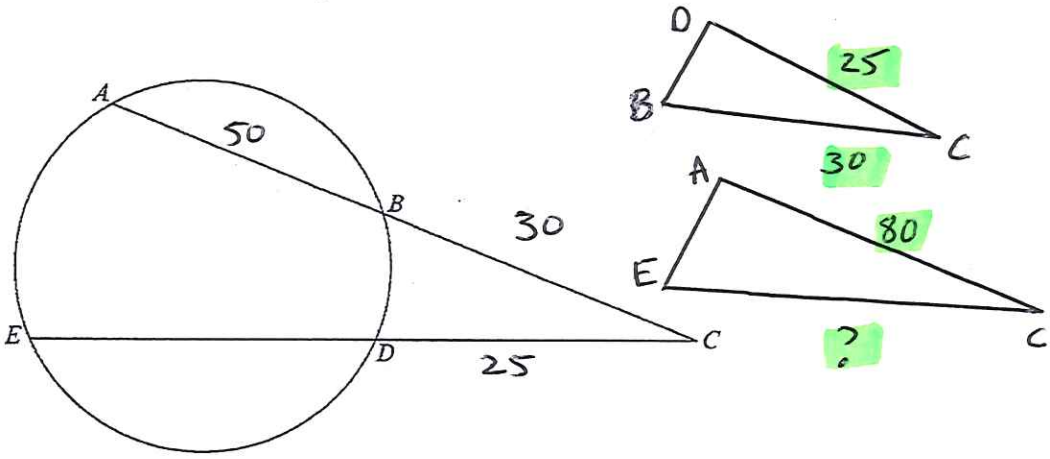


Diagram is not drawn to scale

Calculate the length of ED.

$\times \frac{80}{25}$

CD → CA

3·2

BC → EC

×3·2

EC = 3·2 × 30 = 96

ED = 96 − 25 = 71cm

[4]

14.

The diagram shows a circle with centre O and a tangent PT that touches the circle at C.

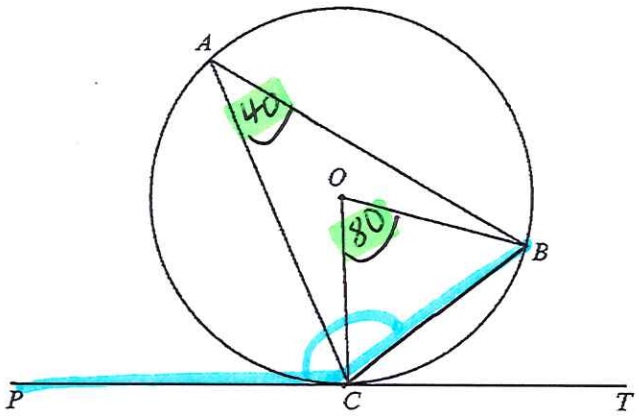


Diagram not drawn to scale

The reflex angle at the centre of the circle is 280°.
Find the size of each of the following angles.
You must give a reason for each answer.

(a) $B\hat{A}C$ acute angle at centre = 80°

$B\hat{A}C = \frac{1}{2}$ of 80° = 40°

(Angle at centre is twice angle at circumference) [2]

(b) $B\hat{C}P$

$B\hat{C}P = P\hat{C}O + O\hat{C}B$

$P\hat{C}O = 90°$ (angle between tangent & radius = 90°)

$O\hat{C}B = (180-80) \div 2 = 50°$ (isosceles triangle)

$B\hat{C}P = 90° + 50° = 140°$

[3]

15.

(a)

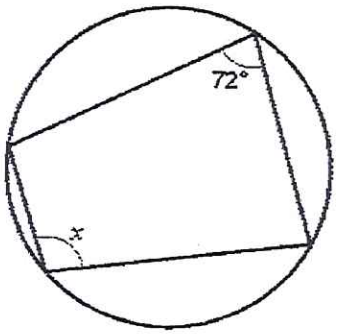


Diagram not drawn to scale

Calculate the size of the angle x in the diagram above. [1]

Opposite angles in a cyclic quadrilateral add upto 180°

$x = 108$ °

(b) The diagram below shows a circle with centre O.
A, B and C are points on the circumference of the circle.
The tangent, PAH, touches the circle at A.
OBH is a straight line.

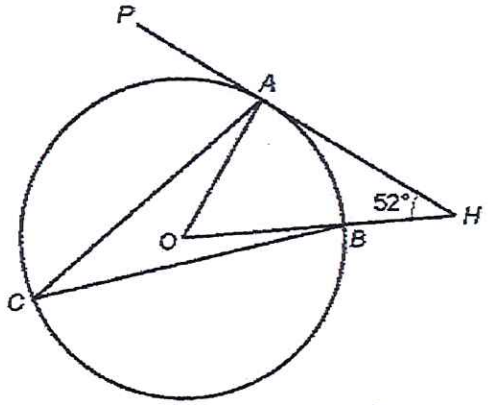


Diagram not drawn to scale

Given that $A\hat{H}B = 52°$, calculate $A\hat{C}B$.
You must show your working. [3]

$H\hat{A}O = 90°$ (angle between tangent & radius = 90°)

$A\hat{O}H = 180 - (90 + 52) = 38°$ (angle sum of triangle)

$A\hat{C}B = \frac{1}{2}$ of $38°$ (angle at centre = 2 x angle at circumference)

$A\hat{C}B = 19$ °

16. The diagram shows a circle with centre O.
The straight lines RT and ST are tangents to the circle, meeting the circle at B and C respectively.

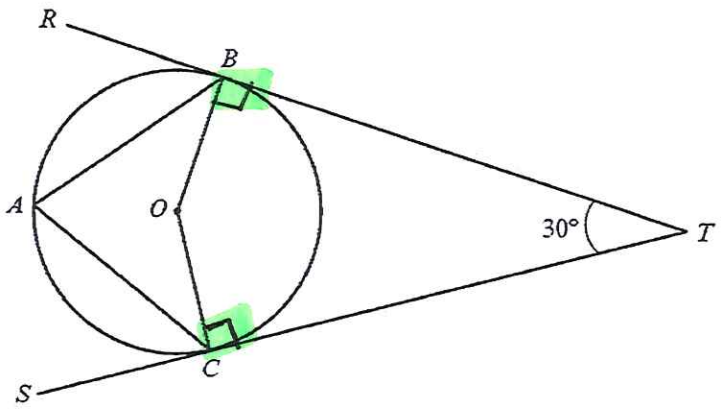


Diagram not drawn to scale

Given that $B\hat{T}C = 30°$, calculate the size of $B\hat{A}C$.
You must give reasons in your solution.

$O\hat{B}T = O\hat{C}T = 90°$ (angle between tangent and radius = 90°)

$B\hat{O}C = 360 - (90+90+30)$ (angles in a quadrilateral add upto 360°)
$= 150°$

$B\hat{A}C = 75°$ (angle at centre = 2 × angle at circumference)

[4]

17.

The points A, B, C and D lie on the circumference of a circle with centre O.
All the angles are measured in degrees.

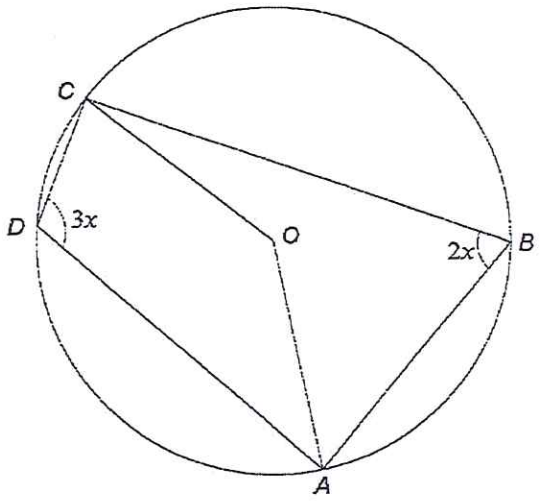


Diagram not drawn to scale

Find the size of the obtuse angle $A\hat{O}C$. [4]

$3x + 2x = 180°$

$5x = 180°$

$x = 180/5 = 36°$ (opposite angles in a cyclic quadrilateral total 180°)

$A\hat{O}C = 4x$ (angle at centre = 2× angle at circumference)

$A\hat{O}C = 4 \times 36°$

$A\hat{O}C = 144°$

18.

(a)

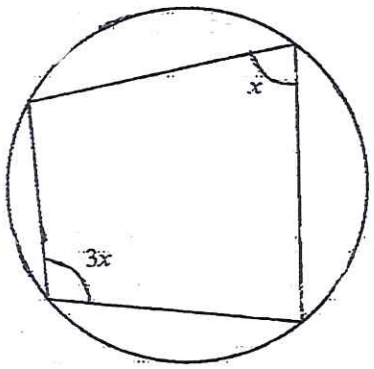


Diagram not drawn to scale

Find the size of angle x. [2]

$x + 3x = 180°$ (opposite angles in a cyclic quadrilateral total 180°)

$4x = 180°$

$x = 180/4$

$x = 45°$

(b) In the diagram, O is the centre of the circle and OB is parallel to the tangent at C.

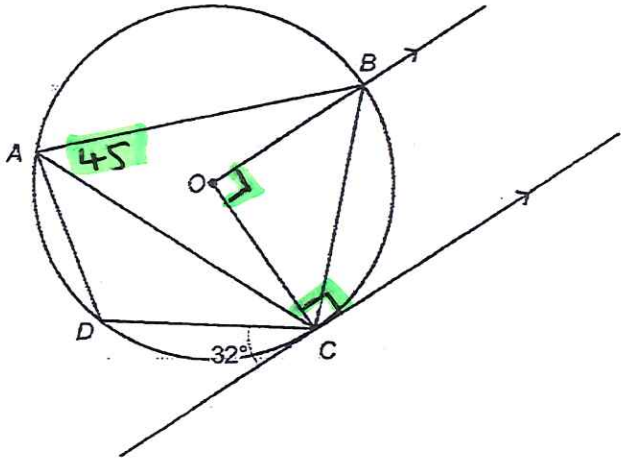


Diagram not drawn to scale

Calculate the size of $\hat{DAB}$.
You must indicate any angles you calculate. [4]

$O\hat{C}B = 90°$ (angle between radius & tangent = 90°)

$B\hat{O}C = 90°$ (interior angles parallel lines)

$B\hat{A}C = 45°$ (angle at centre = 2× angle at circumference)

$D\hat{A}C = 32°$ (alternate segment theorem)

$D\hat{A}B = D\hat{A}C + B\hat{A}C = 45 + 32 = 77$

$\hat{DAB} = 77°$

19. The points A, B and C lie on the circumference of a circle, centre O. AD is a tangent to the circle. DCB is a straight line.

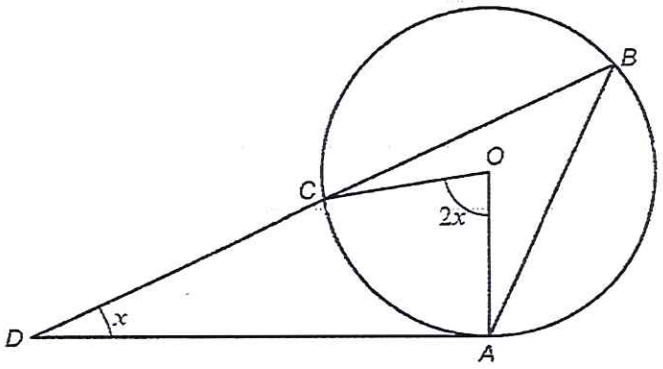


Diagram not drawn to scale

Find the size of each of the following angles, in terms of x.
Write your answers in their simplest form.

(a) $O\hat{C}D$, [2]

$O\hat{A}D = 90°$ (angle between radius & tangent is a right angle)

$O\hat{C}D = 360 - (90 + 2x + x)$ (angles in a quadrilateral total 360°)

$O\hat{C}D = 270 - 3x$

(b) $O\hat{A}B$. [3]

$A\hat{B}C = x$ (angle at centre = 2x angle at circum.)

$D\hat{A}B = 180 - 2x$ (angles in a triangle add up to 180° in ΔDAB)

$D\hat{A}O = 90°$ (angle between tangent & radius)

$O\hat{A}B = D\hat{A}B - D\hat{A}O = 180 - 2x - 90 = 90 - 2x$

20. *In this question, you will be assessed on the quality of your organisation, communication and accuracy in writing.*

Points *A*, *B*, *C* and *D* lie on the circumference of a circle, centre *O*.
BD is a diameter of the circle.
The straight line $BC = 4{\cdot}7$ cm and $\hat{BAC} = 28°$.

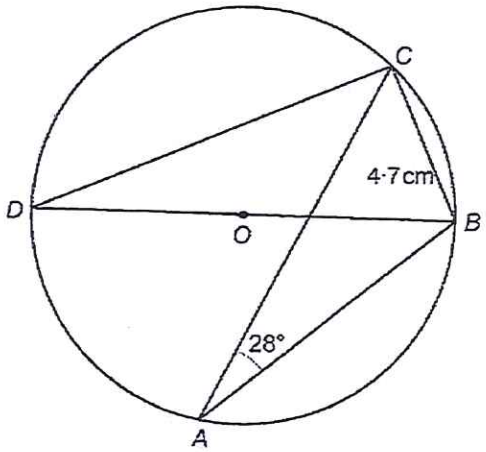
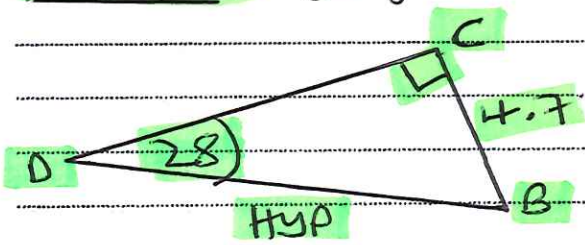


Diagram not drawn to scale

Write down the size of $\hat{BDC}$.
Hence, calculate the length *BD*.
You must show all your working. [5 + 2 OCW]

$B\hat{D}C = 28°$ (angles in the same segment)

$B\hat{C}D = 90°$ (angle in a semicircle = 90°)



$\sin 28 = \frac{OPP}{HYP}$

$\sin 28 = \frac{4.7}{HYP}$

$DB = HYP = \frac{4.7}{\sin 28}$

$DB = 10.01$ cm (2dp)