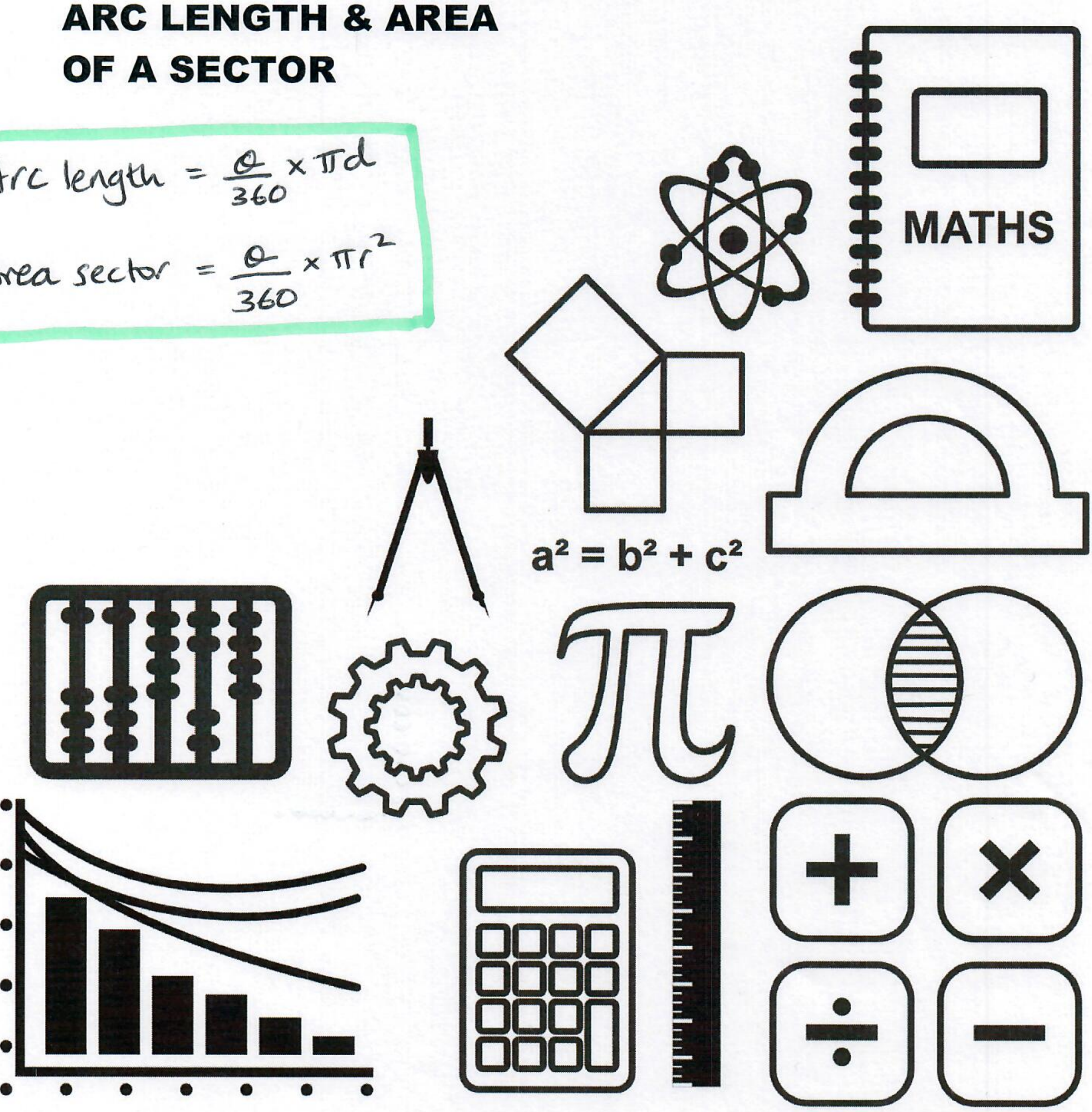
MATHSDIY

SOLUTIONS

GCSE TOPIC BOOKLET ARC LENGTH & AREA OF A SECTOR

Arc length $= \frac{\theta}{360} \times \pi d$

Area sector $= \frac{\theta}{360} \times \pi r^2$



1.

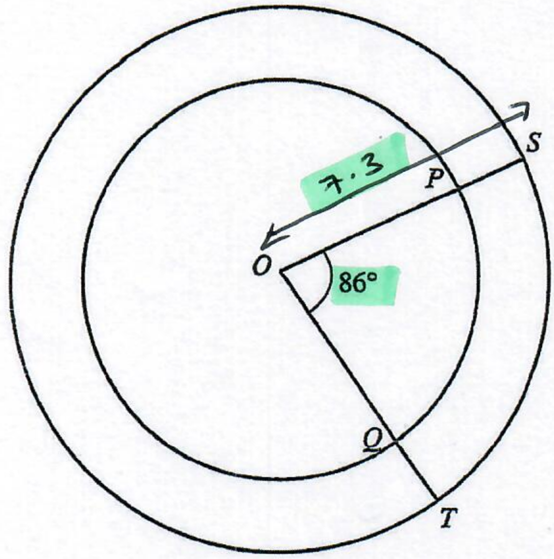


Diagram not drawn to scale.

The diagram shows two concentric circles with centre O.
OQ and OP are radii of the smaller circle.
OS and OT are radii of the larger circle.
The radius of the larger circle is 7·3 cm.
$P\hat{O}Q = 86°$.

(a) Calculate the area of sector SOT.

$$= \frac{\theta}{360} \times \pi r^2$$

$$= \frac{86}{360} \times \pi \times 7.3^2$$

$$= 39.9936.. = \underline{40.0\,\text{cm}^2}$$

[2] (1dp)

(b) The area of the sector POQ is $20{\cdot}3\,\text{cm}^2$. Calculate the radius of this sector POQ.

$$20.3 = \frac{86}{360} \times \pi \times r^2$$

$$\frac{20.3 \times 360}{86 \times \pi} = r^2$$

$$r^2 = 27.0489..$$

$$r = \sqrt{27.0489..}$$

[3]

$$r = 5.2008..$$

<u>radius of sector POQ = 5.2 cm (1dp)</u>

© MathsDIY.com

2. (a) A circular flower bed in a town park has a radius of 8 metres.
The perimeter of a major sector of this flower bed is marked out with a thin metal strip, as shown below.

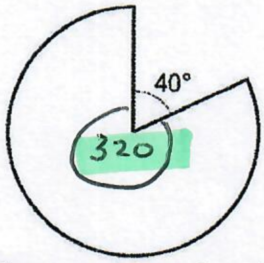


Diagram not drawn to scale

Calculate the total length of the thin metal strip. [3]

Arc length $= \frac{320}{360} \times \pi \times d$

$= \frac{320}{360} \times \pi \times 16 = 44.68042\cdots m$

$= 44.68m$

Total length $= 44.68 + 8 + 8 = 60.68m$

(b) A different circular flower bed has a radius of 12 metres.
The park gardener wants to create a sector *AOB* of this circle that has a surface area of $93\,m^2$.

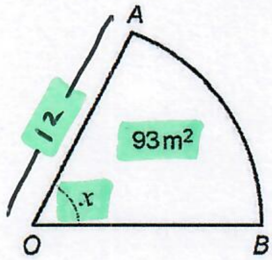


Diagram not drawn to scale

Calculate the size of angle x. [3]

Area of sector $= \frac{\theta}{360} \times \pi r^2$

$93 = \frac{x}{360} \times \pi \times 12^2$

$\frac{93 \times 360}{\pi \times 144} = x$, $x = 74.007\cdots$

$x = 74°$

3. A company logo is made up of three parts: a square and two identical sectors attached to two of its sides, as shown below.

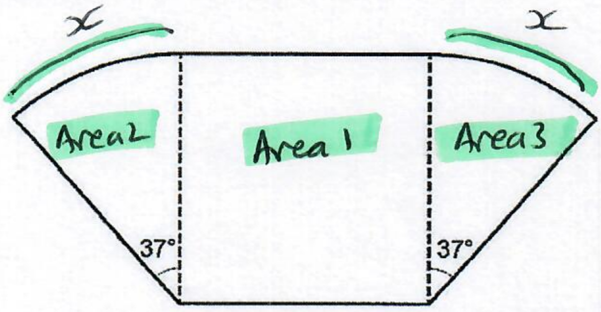


Diagram not drawn to scale

The logo displayed outside the company's head office has a central square with a side length of 7 metres.

(a) Calculate the total area of this logo. [3]

Area1 $= L \times W = 7 \times 7 = 49\text{m}^2$

Area2 $= \dfrac{37}{360}\pi r^2 = \dfrac{37}{360} \times \pi \times 7^2 = 15.8214\ldots\text{m}^2$

Area3 $=$ Area2 $= 15.8214\ldots\text{m}^2$

Total area of logo $= 49 + (15.8214 \times 2) = 80.6428$

$= 80.64\text{m}^2$ (2dp)

(b) Calculate the total length of the perimeter of this logo. [3]

arc length 'x' $= \dfrac{37}{360}\pi d$

$= \dfrac{37}{360} \times \pi \times 14 = 4.5204\ldots\text{m}$

Total length of perimeter of logo

$= (4 \times 7) + (2 \times 4.5204) = 37.04\text{m}$

4. The shaded part of the diagram below shows the top surface of an engine part.

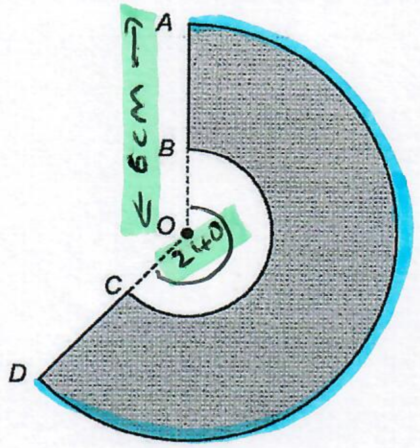


Diagram not drawn to scale

The measurements taken by a motor engineer are:

- reflex angle $B\hat{O}C = 240°$,
- $AO = OD = 6\,\text{cm}$,
- $BO = OC = 3\,\text{cm}$.

(a) The length of the major arc AD is to be sealed by attaching a flexible anti-rust strip.
Each flexible anti-rust strip is of length 35 cm.
What length of the anti-rust strip will be left over after sealing the length of the major arc AD?
Give your answer in terms of π, in its simplest form. [3]

length of major arc AD $= \dfrac{\theta}{360} \pi \times d$

$= \dfrac{240}{360} \times \pi \times 12$

$= \dfrac{2}{3} \times 12\pi = 8\pi$

$\therefore$ rust strip left over $= 35 - 8\pi$

Length of anti-rust strip left over = $35 - 8\pi$ cm

(b) The top surface of the engine part is to be painted.
The paint costs 15p per cm^2.

(i) Calculate the cost of the paint to be used.
Give your answer in terms of π, in its simplest form. [4]

Area sector AOD $= \frac{240}{360} \pi \times 6^2 = \frac{2}{3} \pi \times 36$

$= 24\pi \text{ cm}^2$

Area sector BOC $= \frac{240}{360} \pi \times 3^2 = \frac{2}{3} \pi \times 9$

$= 6\pi \text{ cm}^2$

$\therefore$ Area of top surface $= 24\pi - 6\pi = 18\pi \text{cm}^2$

Cost of paint $= 18\pi \times 15 = 270\pi \text{ p}$ or £2.7π

(ii) Using $\pi = 3$, calculate how much it costs to paint the top surface of 20 engine parts.
Give your answer in pounds. [1]

$2.7 \times \pi \times 20$

$= 54 \times 3$ (using $\pi = 3$)

$= 162$

Paint cost is £ 162

MATHSDIY

5. Points E and F lie on a circle, centre O.
The radius of the circle is 10 cm.
The area of the shaded sector is 65 cm^2.

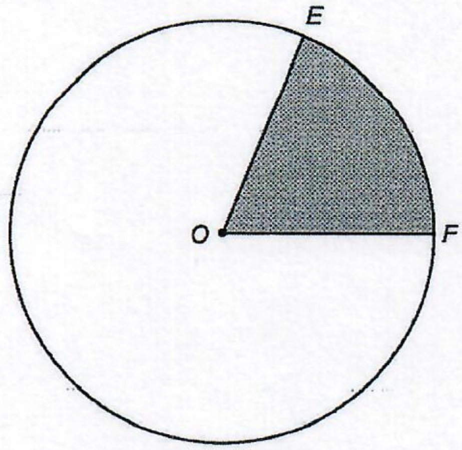


Diagram not drawn to scale

(a) Calculate the size of $E\hat{O}F$. [3]

Area sector EOF $= \frac{\theta}{360}\pi r^2$

$65 = \frac{\theta}{360} \times \pi \times 10^2$

$\frac{65 \times 360}{\pi \times 100} = \theta$, $\theta = 74.4845..$

$\theta = 74.48°$ (2dp)

(b) Hence, calculate the length of the arc EF. [2]

Arc length $= \frac{\theta}{360}\pi d$

$= \frac{74.4845}{360} \times \pi \times 20$

$= 13$ cm

© MathsDIY.com

6.

A sector of a circular metal plate centre O, with radius 18 cm, is removed to leave the following shape.

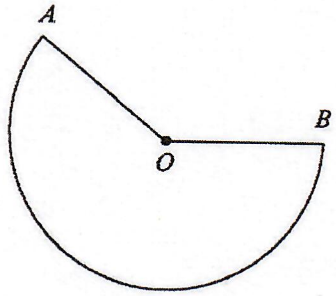


Diagram not drawn to scale

radius = 18cm

diameter = 36cm

The length of the arc AB of the shape is 66 cm.

(a) Calculate the size of the reflex angle AOB correct to the nearest degree.

$$\text{Arc length} = \frac{\theta}{360} \times \pi d$$

$$66 = \frac{\theta}{360} \times \pi \times 36$$

$$\frac{66 \times 360}{\pi \times 36} = \theta \qquad \theta = 210.0845\ldots$$

$$\underline{\theta = 210^\circ}$$

[3]

(b) What was the area of the piece of metal that was removed?

sector removed must have an angle of

$360 - 210 = 150^\circ$

$$\text{Area sector} = \frac{\theta}{360} \times \pi r^2$$

$$\underline{\text{Area sector}} = \frac{150}{360} \times \pi \times 18^2$$

$$= 424.115\ldots$$

[2]

$$= \underline{424.12\ \text{cm}^2} \quad (2dp)$$

© MathsDIY.com

7.

The diagram shows a circular flower bed, which is split into two sectors, one for spring flowers and the other for roses.
The centre of the circle is O and the area of the minor sector is $31{\cdot}3\ \text{m}^2$.

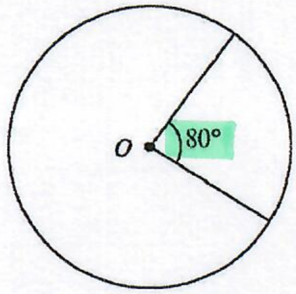


Diagram not drawn to scale

(a) Calculate the radius of the flower bed.

$$\text{Area of sector} = \frac{\theta}{360} \times \pi r^2$$

$$31{\cdot}3 = \frac{80}{360} \times \pi r^2$$

$$\frac{31{\cdot}3 \times 360}{80\pi} = r^2, \quad r^2 = 44{\cdot}8339\ldots$$

$$r = \sqrt{44{\cdot}8339\ldots} \quad r = 6{\cdot}695\ldots$$

$$\underline{r = 6{\cdot}7\,\text{cm}} \quad (1\text{dp})$$ [3]

(b) Calculate the perimeter of the major sector of the flower bed.

Angle of major sector $= 360 - 80 = 280^\circ$

$$\text{Arc length} = \frac{\theta}{360} \times \pi d$$

$$\underline{\text{Arc length}} = \frac{280}{360} \times \pi \times (6{\cdot}695 \times 2) = 32{\cdot}717\ldots$$

$$= \underline{32{\cdot}7\,\text{cm}}$$

$$\underline{\text{Perimeter}} = 6{\cdot}7 + 6{\cdot}7 + 32{\cdot}7$$

$$= \underline{46{\cdot}1\,\text{cm}}$$ [3]

(examiner accepts answers in the range $46{\cdot}0 - 46{\cdot}15$ cm)

8.

A gardener is marking out the border of a flowerbed.
The flowerbed is in the shape of a sector *AOB* of a circle centre *O* as shown below.

The complete border is 28 metres long. → arc length $= 28 - 8{\cdot}6 - 8{\cdot}6$
OA = *OB* = 8·6 m. $= 10{\cdot}8\text{m}$

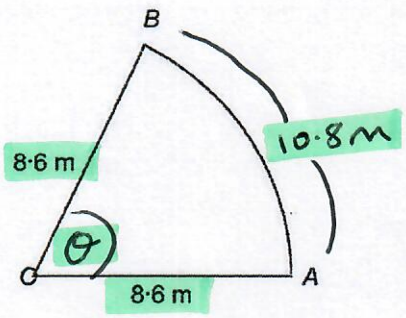


Diagram not drawn to scale

(a) Calculate the size of $A\hat{O}B$. [4]

$\text{arc length} = \dfrac{\theta}{360} \times \pi d$

$10 \cdot 8 = \dfrac{\theta}{360} \times \pi \times (8 \cdot 6 \times 2)$

$\dfrac{10 \cdot 8 \times 360}{\pi \times 17 \cdot 2} = \theta$

$\theta = 71.9528 \ldots$

$\underline{\underline{\theta = 72^\circ}}$

9.

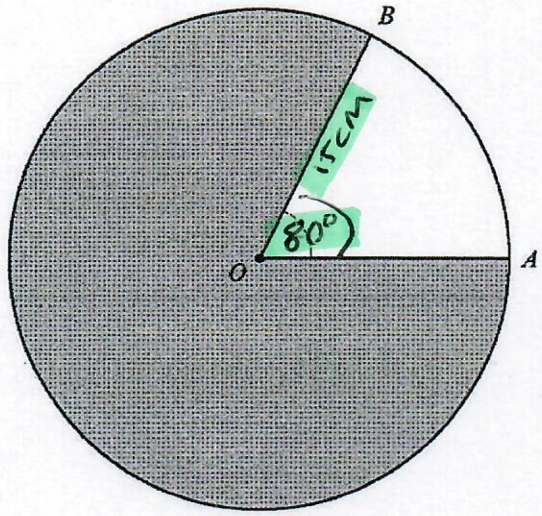


Diagram not drawn to scale

The points A and B lie on a circle with centre O.
The radius of the circle is 15 cm and $A\hat{O}B = 80°$.

(a) Calculate the length of the minor arc AB.

arc length $= \dfrac{\theta}{360} \times \pi d$

$= \dfrac{80}{360} \times \pi \times (2 \times 15)$

$= 20.94 39\ldots = \underline{\underline{20.9\text{ cm}}}$ (1dp)

[2]

(b) Calculate the area of the shaded sector of the circle.

$\theta = 360 - 80 = 280°$

area sector $= \dfrac{\theta}{360} \times \pi r^2$

$= \dfrac{280}{360} \times \pi \times 15^2$

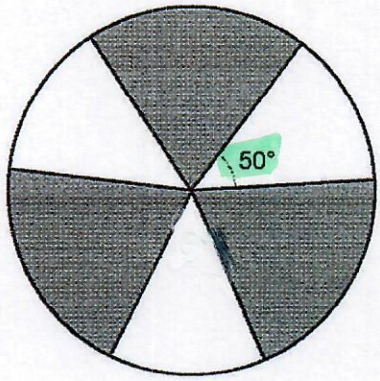
[3]

$= 549.7787\ldots$

$= \underline{\underline{549.8\text{ cm}^2}}$ (1dp)

10.

A circular logo, with radius 8 cm, is shown below.



All three white sectors are equal in size and shape.
All three shaded sectors are equal in size and shape.

(a) Calculate the total area of the shaded sectors. [4]

White sectors have a total angle of $3 \times 50 = 150°$

Shaded sectors have a total angle of $360 - 150 = 210°$

Angle of 1 shaded sector $= 210 \div 3 = 70°$

area shaded sector $= \frac{70}{360} \times \pi \times 8^2$

$= 39.095\ldots$

Total shaded area $= 3 \times 39.095\ldots = 117.3 \text{ cm}^2$ (1dp)

(b) The whole perimeter of all the shaded sectors is to be drawn in red.
Calculate the total length of all these red boundary lines. [4]

1 arc length $= \frac{70}{360} \times \pi d = \frac{70}{360} \times \pi \times 16 = 9.7738\ldots$

3 arc lengths $= 29.3215\ldots$

Total red boundary $= 6 \times \text{radius} + 29.3215$

$= 6 \times 8 + 29.3215$

$= 77.3 \text{ cm}$ (1dp)

11.

The diagram shows two sectors of circles, *BOC* and *AOD*.
The circles have a common centre, *O*.
You are given that $A\hat{O}D = 131°$, $OB = OC = 8{\cdot}4$cm and $BA:AO$ is 1:2.

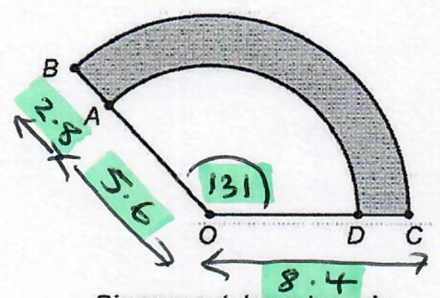


Diagram not drawn to scale

Calculate the perimeter of the shaded region *ABCD*. [6]

$BA = \frac{1}{3} OB \qquad BA = \frac{8.4}{3} = 2.8\text{cm}$

$AO = 8.4 - 2.8 = 5.6\text{cm}$

$\text{Arc } AD = \frac{131}{360} \times \pi d = \frac{131}{360} \times (5.6 \times 2) \times \pi = 12.8037\ldots$

$\text{Arc } BC = \frac{131}{360} \times \pi d = \frac{131}{360} \times (8.4 \times 2) \times \pi = 19.2056\ldots$

$\text{Perimeter of } ABCD = 19.2056 + 12.8037 + (2 \times 2.8)$

$= 37.6093$

$= 37.6\text{cm}$ (1dp)

Perimeter of *ABCD* is 37·6 cm

12.

The radius of the circle below is 3·6 cm.

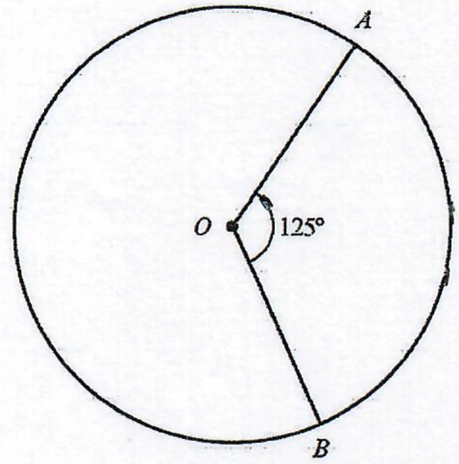


Diagram not drawn to scale

Calculate the length of the minor arc AB.

arc length $= \frac{\theta}{360} \times \pi \times d$

$= \frac{125}{360} \times \pi \times (3 \cdot 6 \times 2)$

$= 7 \cdot 8539 \ldots$

[2]

$= \underline{\underline{7 \cdot 85\text{cm}}}$ (2dp)

13. A landing area, as shown below, is marked out for a throwing event in a sports field.
AB is an arc of a circle centre O. The angle $A\hat{O}B = 60°$ and $OA = OB = 80$m.

radius

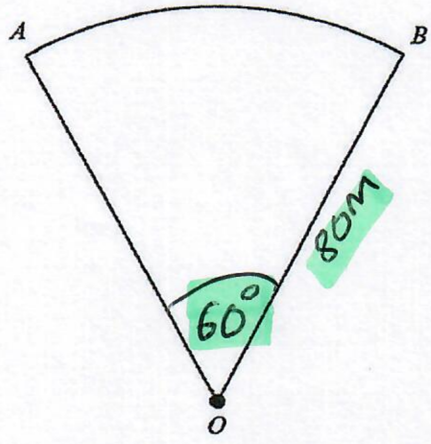


Diagram not drawn to scale

A rope is used to mark the boundary of the whole landing area.

(a) Calculate the area enclosed by the rope.

$$\text{Area} = \frac{60}{360} \times \pi \times 80^2$$

$$= 3351.032\ldots$$

$$= \underline{\underline{3351.0 \text{ m}^2}} \quad (1\text{dp})$$

[2]

(b) What is the total length of the rope that is used to mark the landing area?

$$= 80 + 80 + \text{arc } AB$$

$$= 80 + 80 + \frac{60}{360} \pi \times (2 \times 80)$$

$$= 160 + 83.775\ldots$$

$$= \underline{\underline{243.8 \text{ m}}} \quad (1\text{dp})$$

[4]

© MathsDIY.com